POLYNOMIALLY CONVEX SETS

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0. **Introduction.** This is a report on the *polynomial convex hull* of a compact set, X, in complex n-space, C^n. By definition, it is hull(X), *the set of all p in C^n such that*

$$|f(p)| \leqq \max_{x \in X} |f(x)|,$$

for every polynomial, $f(z_1, \cdots, z_n)$. When $X = \text{hull}(X)$, we say *X is polynomially convex*. In studying the polynomial convex hull, it helps to introduce also R-hull(X), the *rational convex hull* of X. By definition, R-hull(X) is *the set of all p in C^n such that*

$$|g(p)| \leqq \max_{x \in X} |g(x)|,$$

for all rational functions, g, which are analytic about X. Equivalently, R-hull(X) may be described as the set of all p in C^n for which $f(p)$ belongs to $f(X)$, for every polynomial, f. When $X = R\text{-hull}(X)$, we say that *X is rationally convex*. Evidently,

$$X \subset R\text{-hull}(X) \subset \text{hull}(X),$$

and these hulls are compact.

Polynomially convex sets occur prominently in the theory of uniform approximation. Every finitely-generated function algebra can be realized as the uniform closure of the polynomials on a compact subset, X, of some C^n; in which case, its maximal ideal space is precisely hull(X) [**13**].

I. **Local descriptions of the hulls.** To begin, we explain what we mean by a curve of analytic hypersurfaces in an open subset, O, of C^n. If U is a domain in O and F_t, $0 \leqq t < 1$, is a curve of nonconstant analytic functions on U, we let H_t be the zero-set of F_t in U. If each H_t is closed in O, then we say that *(F_t, H_t) is a curve of analytic hypersurfaces in O*. We shall denote it, simply, by (H_t).

Almost all the results of §§I and II are based on the following beautiful local characterization of the polynomial convex hull, given by K. Oka in 1937, in [6].

(I.0) Oka's characterization theorem. *Let O be a neighborhood of* hull(X) *in C^n. If (H_t) is a curve of analytic hypersurfaces in O such*