BOOK REVIEWS

Integral operators in the theory of linear partial differential equations. By Stefan Bergman. Springer, Berlin, 1961. 8+145 pp. DM39.80.

The report summarizes the recent development in the theory of integral operators for generating solutions of linear partial differential equations from complex analytic functions. The intention of this approach is the development of parts of the theory of those equations on the basis of complex analysis, and the use of the operators may be regarded as a "translation principle" under which certain theorems on analytic functions carry over into theorems which characterize various general properties of those solutions. In this respect the operator theory seems to be a valuable addition to the classical theory which concentrates on existence and uniqueness problems. The author uses special operators which preserve properties such as the validity of series developments and the connection between the coefficients of these series and the location and character of singularities.

The consideration starts with operators $P(f)$ for generating solutions u of partial differential equations in two variables. Here $f(z)$ is an analytic function, called the associate of the solution u. Various representations of the so-called Bergman operator of the first kind are given. This operator is particularly useful for treating the coefficient problem, because it has a very simple inverse. Another important operator considered in the first chapter is the integral operator of exponential type, which generates solutions satisfying ordinary differential equations, in addition to the partial differential equation.

The next chapter is devoted to the Whittaker-Bergman operator for generating harmonic functions of three variables. Choosing the associates in a systematic fashion the author obtains a classification of the corresponding harmonic functions and discusses general properties enjoyed by all the functions of each such class.

In Chapter 3 it is shown that more general solutions of differential equations in three independent variables can be treated by operator methods. In fact, there exist operators transforming harmonic functions of three variables into those solutions. The consideration concentrates on those operators which preserve certain properties of the harmonic functions involved.

The application of the operators to systems of partial differential equations is still in its initial stage. An introduction to this part of the theory is given in Chapter 4, and it is shown that some of the oper-