SINGULARITIES OF PIECEWISE LINEAR MAPPINGS. I MAPPINGS INTO THE REAL LINE

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In this note we consider piecewise linear mappings of combinatorial manifolds into the real line. We define—in a purely combinatorial way—a certain class of mappings called nondegenerate and we prove that every continuous mapping may be approximated by a nondegenerate one. Nondegenerate mappings on a combinatorial manifold behave like differentiable nondegenerate mappings on a differentiable manifold. In particular, the index of a singularity can be defined and one can prove the Morse inequalities and an analogue of the Reeb theorem about a function with only two nondegenerate singularities.

The approximation theorem can also be extended to height functions on combinatorial submanifolds of Euclidean space.

Detailed proofs will be published later. We give here descriptions of the singularities, a statement of the main theorems and a very brief sketch of the approximation theorem.

1. E^n will denote n-dimensional Euclidean space. By a complex we shall understand a rectilinear, locally finite simplicial complex in Euclidean space. A closed subspace $M \subset E^n$ will be said to be a manifold if M is a space of a complex K which is a connected combinatorial manifold in the sense of M.H.A. Newman and J. W. Alexander (see [1]). We will then say that K is a triangulation of M. The boundary $\dot{M}$ of M will be the subset of M covered by the mod 2 boundary of its triangulation; if $\dot{M} = \emptyset$ we will say that M is unbounded, a compact and unbounded manifold will be said to be closed.

Let U, V be open subsets of a manifold M and let f: $V \rightarrow M_1$ be a continuous mapping of V into a manifold M_1. We say that f is *piecewise linear* (shortly: PL) in U if there exist triangulations K of M and K_1 of M_1 such that for every simplex $\triangle$ of K intersecting U, f maps $\triangle \cap V$ linearly into a simplex of K_1. If $U = V = M$ we will say that f is a PL mapping of M. If $p \in V \subset M$ and f: $V \rightarrow M_1$ is PL in a certain neighborhood of p then we will say that f is PL at p.

Let f_i: $M_i \rightarrow E^k$ be two PL-mappings, and let $p_i \in M_i$, $i = 1, 2$. We say that f_1 at p_1 is *equivalent* to f_2 at p_2 if there exist neighborhoods U_i of p_i, $i = 1, 2$ and homeomorphisms h: $E^k \rightarrow E^k$, g: $U_1 \rightarrow U_2$ such that

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