RESEARCH PROBLEMS

26. P. T. Church and E. Hemmingsen: *Topology.*

Let $f\colon M \to N$ be continuous, where M and N are n-manifolds, $n \geqq 3$. Let the branch set B_f of f be the set of points at which f is not a local homeomorphism. Can $\dim(B_f) = 0$?

If it is also assumed that $\dim(f(B_f)) < n$, the question can be reduced (by appropriate restriction of the map) to the following: Does there exist such an f for which $f|f^{-1}(f(B_f))$ (restriction) is one-to-one, $f|M - f^{-1}(f(B_f))$ is a k-to-one covering map for some natural number k, $N = E^n$, and $M \subset E^n$? (Cf. Duke Math. J. **27** (1960), 527–536; an example of a three-to-one map $f\colon S^n \to S^n$ for which B_f has [some] point components appears in the same journal **28** (1961).) (Received October 19, 1961.)

27. Richard Bellman: *Calculus of variations—differential equations.*

The theory of stability of solutions of differential equations is concerned with the invariance of properties of solutions under structural changes in the equations. In particular, a great deal of effort has gone into the study of boundedness of solutions. Frequently, however, precise bounds have not been obtained. Towards this end, it would be worthwhile to obtain solutions to the following questions:

Given the linear differential equation $u'' + (1 + f(t))u = 0$, $u(0) = 1$, $u'(0) = 0$, and the associated functional $J(f) = \max_{0 \leq t \leq T} |u(t)|$, determine the maximum of $J(f)$ over all f subject to the constraints

$$\text{(a)} \qquad |f(t)| \leqq b_1 < 1, \qquad 0 \leqq t \leqq T,$$

or

$$\text{(b)} \qquad \int_0^T |f(t)|\, dt \leqq b_2 < \infty,$$

or

$$\text{(c)} \qquad \int_0^T |f(t)|^p dt \leqq b_3 < \infty, \qquad p > 1.$$

Similarly, we would like to determine the maximum of

$$J_1(f) = \int_0^T |u(t)|^q dt, \qquad q \geqq 1,$$

and the asymptotic behavior of these maxima as $T \to \infty$.