BOOK REVIEWS

Abelian groups. By L. Fuchs. Budapest, Publishing house of the Hungarian Academy of Sciences, 1958. 367 pp.

This encyclopedic work on the theory of abelian groups will certainly further stimulate the interest in this subject which has been growing steadily since the publication of Part II of Kuroš' Group theory (1953) and Kaplansky's Infinite abelian groups (1954). This is the first book which includes, along with the fundamental structure theory which appears in the earlier mentioned works, the large body of material contributed in the last several years by the Hungarian school of group theorists. The material in Kuroš is entirely covered. The generalizations to modules over a principal ideal domain and a complete discrete valuation ring, as well as the applications to topological groups, which are features of Kaplansky's monograph, are not included except in the exercises. The author believes that a module-theoretic treatment would imply either almost trivial generalizations and unnecessary complications in the discussions, or deeper extensions which are of a ring-theoretic rather than a grouptheoretic nature. The author has been eminently successful in giving a complete, detailed, and easily understandable account of the present status of the theory of abelian groups with special emphasis on results concerning structure problems.

Chapter I includes a summary of the basic concepts of group theory which appears in any textbook on modern algebra, and the notation and terminology is established. The types of groups which play a central role in the structure theory are described in detail. The rank r(G) of a group G is defined as the cardinal number of a maximal independent set (in G) containing only elements of infinite and prime power order. This definition includes those given by other authors for torsion free groups and p-groups, and the proof of the invariance of r(G) clarifies the relations between these definitions. It should be noted that Prüfer's definition of rank which has been followed by many authors is equivalent to what is here defined as the reduced rank.

Chapter II is devoted to direct sums of cyclic groups and includes the Fundamental Theorem on finitely generated groups, Kulikov's criterion for a p-group to be a direct sum of cyclic groups, and some recent results of Kertész, Szele, and the author giving criteria for the existence of a basis.

Chapter III discusses the properties and structure theory of divisible groups. An alternate proof of the direct summand property of divisible groups is obtained, using Gacsályi's results on the solution of linear equations over divisible groups.

Chapter IV is an exposition of direct summands and pure sub-