MATRICES OF ZEROS AND ONES

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Let A be a matrix of m rows and n columns and let the entries of A be the integers 0 and 1. We call such a matrix a *(0, 1)-matrix of size m by n*. The 2^{mn} (0, 1)-matrices of size m by n play a fundamental role in a wide variety of combinatorial investigations. One of the chief reasons for this is the following. Let X be a set of n elements $x_1, x_2, \cdots, x_n$ and let $X_1, X_2, \cdots, X_m$ be m subsets of X. Let $a_{ij}=1$ if x_j is a member of X_i and let $a_{ij}=0$ if x_j is not a member of X_i. The a_{ij}'s yield a (0, 1)-matrix $A=[a_{ij}]$ of size m by n called the *incidence matrix* for the subsets $X_1, X_2, \cdots, X_m$ of X. The 1's in row i of A specify the elements that belong to set X_i and the 1's in column j of A specify the sets that contain element x_j. The matrix A characterizes the m subsets $X_1, X_2, \cdots, X_m$ of the set X.

Let A be a (0, 1)-matrix of size m by n. Let the sum of row i of A be denoted by r_i and let the sum of column j of A be denoted by s_j. We call

$$R = (r_1, r_2, \cdots, r_m)$$

the *row sum vector* and

$$S = (s_1, s_2, \cdots, s_n)$$

the *column sum vector* of A. If τ denotes the total number of 1's in A, then it is clear that

$$\tau = \sum_{i=1}^{m} r_i = \sum_{j=1}^{n} s_j.$$

The vectors R and S determine a class

$$\mathfrak{A} = \mathfrak{A}(R, S),$$

consisting of all (0, 1)-matrices of size m by n with row sum vector R and column sum vector S. In this paper we summarize portions of the extensive literature on (0, 1)-matrices and give special emphasis to problems dealing with the class $\mathfrak{A}(R, S)$. We discuss diversified topics including traces, term ranks, widths, heights, and combinatorial designs. A good deal of the subject matter is still in its infancy

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