FIXED POINT FREE INVOLUTIONS AND EQUIVARIANT MAPS[1]

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1. Preliminaries. We are concerned with involutions without fixed points, together with equivariant maps connecting such involutions. An involution T is a homeomorphism of period 2 of a Hausdorff space X onto itself; that is, $T^2(x) = x$ for all $x \in X$.

There is associated with an involution T on X the orbit space X/T, obtained by identifying x with $T(x)$ for all $x \in X$. Denote by $\nu: X \to X/T$ the decomposition map. If T is fixed point free, then $\nu: X \to X/T$ is a local homeomorphism. The map ν is always both open and closed.

In addition to fixed point free involutions we also study equivariant maps. If spaces X, X' carry involutions T, T' respectively, then a map $m: X \to X'$ is *equivariant* provided that $m(T(x)) = T'(m(x))$ for all $x \in X$. An equivariant map m: $X \to X'$ induces a map M: $X/T \to X'/T'$.

The most fundamental involution without fixed points is the antipodal involution A on the n-sphere S^n, given by $A(x_1, \cdots, x_{n+1}) = (-x_1, \cdots, -x_{n+1})$. When we speak of S^n as carrying a fixed point free involution, it is to be understood that we refer to A. For S^n, we have the classical body of results of Lyusternik-Schnirelmann and Borsuk-Ulam. Some of these well-known results are summarized in the following.

(1.1) *The following are true for every n:*

(i) *there is no equivariant map of S^{n+1} into S^n;*

(ii) *every equivariant map of S^n into itself is of odd degree; in particular, it is essential;*

(iii) *the Lyusternik-Schnirelmann category of $S^n/A = P^n$ is n;*

(iv) *for every covering of S^n by $n+1$ closed sets $A_1, \cdots, A_{n+1}$, some set A_i contains an antipodal pair;*

(v) *for every map f: $S^n \to R^n$ there is a point $x \in S^n$ for which $f(x) = f(A(x))$.*

An address delivered before the East Lansing meeting of the Society by P. E. Conner under the title *Involutions and equivariant maps*, on September 2, 1960, by invitation of the Committee to Select Hour Speakers for Summer and Annual Meetings; received by the editors June 24, 1960.

[1] This research was supported by the United States Air Force through the Air Force Office of Scientific Research of the Air Research and Development Command, under contract AF 49(638)-72 at the University of Virginia.