# LINEAR GROUPS OVER LOCAL RINGS

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A *local ring* is a commutative ring $L$ with unit which has a greatest ideal $I \neq L$. The set $L^* = L - I$ of units in a local ring $L$ forms a group under the multiplication. $L/I$ is a field, the so-called residue field of $L$. The homomorphic image of a local ring, if it is not the zero ring 0, is again a local ring.

An *n-dimensional vector space over* $L$, $V_n(L)$, is a $L$-module isomorphic to $L^n$. An *m-dimensional subspace* $W$ *of* $V = V_n(L)$ is a direct summand isomorphic to $L^m$.

The *general linear group in n variables over* $L$, $GL(n, L)$, is the group of linear automorphisms of $V_n(L)$. We propose to study the structure of this group, more precisely, we wish to describe the position of the invariant subgroups of $GL(n, L)$. In the case that $L$ is a field it is well known that $GL(n, L)$ has only big and small invariant subgroups, that is to say, in this case $GL(n, L)$ has only invariant subgroups which either contain the special linear group $SL(n, L)$ or else are contained in the center $Z(GL(n, L))$ of $GL(n, L)$, cf. Dieudonné [3] and [4] and Artin [1]. If $L$ is not a field, however, there will be nontrivial ideals in $L$ which give rise to more invariant subgroups, the so-called congruence subgroups modulo an ideal $J$ of $L$. Our main result is, cf. Theorem 3 below, that for a local ring $L$ it is still possible to get a survey over the different invariant subgroups $G$ of $GL(n, L)$, each of which is determining an ideal $J$ of $L$ such that $G$ is situated between a greatest and a smallest congruence subgroup mod $J$. In the case that this ideal $J$ is $L$ or 0 (the only possibilities if $L$ is a field) these greatest and smallest congruence subgroups are $GL(n, L)$ and $SL(n, L)$ (for $J = L$) and $Z(GL(n, L))$ and $E =$ unit group (for $J = 0$) respectively.

Let $J$ be an ideal of the local ring $L$. Denote by $g_J$ the natural homomorphism $L \to L/J$. By the same letter we denote the natural homomorphism $g_J$: $V_n(L) \to V_n(L/J)$. $g_J$ determines the homomorphism

$$h_J: GL(n, L) \to GL(n, L/J)$$

with $h_J \sigma g_J = g_J \sigma$ for $\sigma \in GL(n, L)$.

Let $J$ be an ideal of $L$. The *general congruence subgroup* mod $J$ *of* $GL(n, L)$, $GC(n, L, J)$, is defined by