CLIFFORD PARALLELS IN ELLIPTIC $(2n-1)$-SPACE AND ISOCLINIC n-PLANES IN EUCLIDEAN $2n$-SPACE[1]

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Communicated by Irving Kaplansky, March 18, 1960

An elliptic space is a projective space turned into a metric space by according a special role to an arbitrarily chosen but fixed non-degenerate imaginary hyperquadric. Let p_1, p_2 be any two points in the elliptic space. Then the distance between p_1, p_2 is defined as $(1/2(-1)^{1/2}) \log (p_1p_2\, q_1q_2)$, where q_1, q_2 are the two points at which the line $p_1 p_2$ intersects the hyperquadric and $(p_1p_2\, q_1q_2)$ denotes the cross-ratio of these four collinear points. It follows at once from the definition that (i) the distance (between any two real points) may be taken to be d or $\pi-d$ with $0 \leqq d \leqq \pi$, (ii) distances on the same straight line are additive, and (iii) the total length of any straight line is π.

It is well known that in an elliptic space of dimension 3, the concept of Clifford parallelism exists which has many interesting properties (see, for example, Klein [5]). A similar concept of parallelism for elliptic spaces of dimension $\geqq 3$ is the concept of Clifford-parallel $(n-1)$-planes in an elliptic space, El^{2n-1}, of dimension $2n-1$. We define this as follows:

In an El^{2n-1}, two $(n-1)$-planes A and B are said to be *Clifford-parallel* if the distance to B from any point in A is the same. The relation between two $(n-1)$-planes of being Clifford-parallel is reflexive, symmetric but not transitive. A set of $(n-1)$-planes in El^{2n-1} is called a *maximal set of mutually Clifford-parallel $(n-1)$-planes* if every $(n-1)$-plane in the set is Clifford-parallel to every other $(n-1)$-plane in the set, and if the set is not a subset of a larger set of mutually Clifford-parallel $(n-1)$-planes. A maximal set of mutually Clifford-parallel $(n-1)$-planes in El^{2n-1} is said to form a *foliation* (*partial foliation*) of El^{2n-1} if through each point of El^{2n-1} there passes one and only one (at most one) $(n-1)$-plane of the set.

Existence of maximal sets of mutually Clifford-parallel $(n-1)$-planes in any El^{2n-1} is established by the following theorem:

THEOREM 1. *In an* $\mathrm{El}^{2n-1}(n>1)$, *there are two or more maximal sets of mutually Clifford-parallel $(n-1)$-planes containing any given $(n-1)$-plane. If n is odd, there exist only* 1-*dimensional*[2] *maximal sets. If*

[1] Some of the results contained in this paper were obtained while the author was participating in a National Science Foundation Research Project at the University of Chicago in 1959.

[2] We call a set of $(n-1)$-planes p-dimensional if it depends on p parameters.