MAXIMUM THEOREMS FOR SOLUTIONS OF HIGHER ORDER ELLIPTIC EQUATIONS

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The classical maximum modulus theorem for solutions of second order elliptic equations was recently extended by C. Miranda [4] to the case of real higher order elliptic equations in two variables. Previously Miranda [3] has derived a maximum theorem for solutions of the biharmonic equation in two variables. In the case of more variables it was observed by Agmon-Douglis-Nirenberg [2] that a maximum theorem holds in the special case of elliptic operators with constant coefficients with no lower order terms when the domain of definition is a half-space.

In this note we describe a very general maximum theorem for solutions of (complex) higher order elliptic equations in any number of variables. We shall obtain various estimates in the maximum norm which will contain as a special case the extension of Miranda's results to any number of variables.

We denote by G a bounded domain in E_n with boundary ∂G and closure $\overline{G}$. For a function $u \in C^j(\overline{G})$ we introduce the usual maximum norm:

$$\|u\|_j^{\overline{G}} = \max_{|\alpha| \leqq j} \max_{x \in \overline{G}} | D^\alpha u(x) | . \tag{1}$$

Here $\alpha = (\alpha_1, \cdots, \alpha_n)$ is a multiple index of length $|\alpha| = \alpha_1 + \cdots + \alpha_n$ and D^α is the corresponding partial derivative. Furthermore, for continuous functions u in $\overline{G}$ we introduce negative maximum norms $\|u\|_{-j}^{\overline{G}}$ $(j > 0)$ defined in the following manner. Write u in the form

$$u = \sum_{|\alpha| \leq j} D^\alpha f_\alpha \tag{2}$$

with $f_\alpha \in C^{|\alpha|}(\overline{G})$. Then:

$$\|u\|_{-j}^{G} = \operatorname{Inf} \max_{|\alpha| \leqq j} \|f_\alpha\|_0^{\overline{G}}, \tag{3}$$

where the infimum is taken over all possible representations of the form (2).

Actually we are going to use negative norms for functions f defined on the (sufficiently smooth) boundary. If f has continuous derivatives