

about the sets introduced by Carleson and Helson which, on account of the questions remaining still open, raise interesting problems for the research student.

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Tauberian theorems. By Harry Raymond Pitt. Tata Institute of Fundamental Research Monographs on Mathematics and Physics, vol. 2, Oxford University Press, 1958. 10+174 pp. 30 sh.

The terminology used in this book is well known. A theorem which asserts that, for the transformation

$$(1) \quad g(u) = \int_{-\infty}^{+\infty} k(u, v)s(v)dv,$$

$s(v) \rightarrow a$ as $v \rightarrow +\infty$ implies $g(u) \rightarrow a$ as $u \rightarrow +\infty$ is an *Abelian* or a direct theorem. A *Tauberian* or an inverse theorem asserts that conversely if $g(u) \rightarrow a$ and if $s(v)$ satisfies some additional condition, the so called Tauberian condition, often of the type that $s(v)$ changes slowly with v , then $s(v) \rightarrow a$. A *Mercerian* theorem is an inverse theorem which holds without a Tauberian condition. A theorem is called *special* if it refers to a specific kernel k , and *general* if it holds for an extensive class of kernels.

Tauberian theorems obtained by N. Wiener some 25 years ago, and subsequent contributions of the author, play a central rôle in the whole theory. They are the main subject of this book. Accordingly, readers will find, for example, little about estimation of Tauberian constants, about Tauberian theorems of function theoretic type, asymptotic theorems, best Tauberian conditions and about application of Banach algebras or of locally convex spaces.

The content is as follows: Chapter I-III contain a discussion of very general Tauberian conditions, of slowly decreasing functions; this is followed by elementary general Tauberian theorems and theorems in which boundedness of $g(u)$ implies that of $s(v)$. Special Tauberian theorems are given for the methods of Cesàro, Riesz, Abel and Borel; in the last two cases the proofs furnish also the corresponding high-indices theorems.

Chapters IV and V constitute the main part of the book. After theorems from harmonic analysis about the properties of an analytic function of the Fourier transform $K(t)$ of $k(t)$, the main theorem is proved: if the kernel of (1) is $k(u-v)$, if $K(t) \neq 0$ and if $s(v)$ is bounded, then $S \leq \epsilon + C(\epsilon)G$, where $S = \limsup |s(v)|$, $G = \limsup |g(u)|$. Other classical Wiener theorems follow easily. Refinements of these are then discussed: Tauberian theorems where $K(t) \neq 0$ is assumed