# ON Γ-EXTENSIONS OF ALGEBRAIC NUMBER FIELDS

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Let $p$ be a prime number. We call a Galois extension $L$ of a field $K$ a Γ-extension when its Galois group is topologically isomorphic with the additive group of $p$-adic integers. The purpose of the present paper is to study arithmetic properties of such a Γ-extension $L$ over a finite algebraic number field $K$. We consider, namely, the maximal unramified abelian $p$-extension $M$ over $L$ and study the structure of the Galois group $G(M/L)$ of the extension $M/L$. Using the result thus obtained for the group $G(M/L)$, we then define two invariants $l(L/K)$ and $m(L/K)$, and show that these invariants can be also determined from a simple formula which gives the exponents of the $p$-powers in the class numbers of the intermediate fields of $K$ and $L$. Thus, giving a relation between the structure of the Galois group of $M/L$ and the class numbers of the subfields of $L$, our result may be regarded, in a sense, as an analogue, for $L$, of the well-known theorem in classical class field theory which states that the class number of a finite algebraic number field is equal to the degree of the maximal unramified abelian extension over that field.

An outline of the paper is as follows: in §1–§5, we study the structure of what we call Γ-finite modules and find, in particular, invariants of such modules which are similar to the invariants of finite abelian groups. In §6, we give some definitions and simple results on certain extensions of (infinite) algebraic number fields, making it clear what we mean by, e.g., an unramified extension, when the ground field is an infinite algebraic number field. In the last §7, we first show that the Galois group $G(M/L)$ as considered above is a Γ-finite module, then define the invariants $l(L/K)$ and $m(L/K)$, and finally prove our main formula using the group-theoretical results obtained in previous sections.

1. **Preliminaries.** 1.1. Let $p$ be a prime number. We shall first recall some definitions and elementary properties of $p$-primary abelian groups.[2]

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[2] For the theory of abelian groups in general, cf. I. Kaplansky, *Infinite abelian groups*, University of Michigan Press, 1954.