SOME STRUCTURAL PROPERTIES OF HAUSDORFF MATRICES

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1. **Definitions.** Let $A=(a_{nk})$ denote an infinite matrix. A is called *conservative* if A has finite norm, $a_k=\lim_{n\to\infty} a_{nk}$ exists for each k, and $\lim_{n\to\infty}\sum_k a_{nk}$ exists. A is called *multiplicative* if A is conservative and $a_k=0$ for each k.

s denotes the space of sequences, m the subspace of bounded sequences, and c the subspace of convergent sequences. E_1 is the field of complex numbers and E_∞ the set of sequences, each of which possesses only a finite number of nonzero terms.

Let x be a fixed sequence. Then $c\oplus x=\{y+x \mid y\in c\}$.

Let $H=(h_{nk})$ denote a Hausdorff matrix generated by a sequence μ. I shall use (H, μ) to denote the convergence domain of H, H_μ to denote the matrix, and $H\sim\mu$ to denote the method.

A matrix $A=(a_{nk})$ is said to be of property P, displacement m (written c^mA is of property P) if, for all $k\geqq m$, a_{nk} possesses property P.

A *corridor* matrix is a matrix with the property that there exists a positive integer r such that $a_{nk}=0$ for all n and k with $|n-k|>r$. The smallest such r denotes the width of A.

2. **Introduction.** Let H denote the set of Hausdorff matrices with finite norm. H coincides with the set of conservative Hausdorff matrices as a result of [**1**, page 256, lines 8–12].

Hille [**2**] denotes the set of all multiplicative Hausdorff matrices by M, and observes that it forms a commutative Banach algebra which is also an integral domain. Hence the concepts of unit, prime, divisibility, associate, multiple, and factor can be defined in M. Hille and Tamarkin [**3**, p. 576; **4**, p. 907] observed that every moment function $\mu(z)$ of the form $\mu(z)=(z-a)/(z+b)$, $\Re(a)>0$, $\Re(b)>0$, is prime in M; i.e., $H\sim\mu$ is not equivalent to convergence, but includes only methods that are equivalent to convergence. Hille mentioned this fact in [**2**, p. 422], and again raised the open question as to whether all primes in M are of this form.

From Hille's definition of a prime moment function, a regular Hausdorff matrix H with the property that $(H, \mu)=c\oplus x$ for some unbounded sequence x would have a moment function $\mu(z)$ which would be a prime element of M. The results stated in this paper show that it is impossible to construct a Hausdorff matrix $H\in$H