*Geometric integration Theory* by Hassler Whitney. Princeton, Princeton University Press, 1957. 15+387 pp. $8.50.

This book deals mainly with the following problem connecting algebraic topology with analysis and differential geometry: Characterize those cochains which, together with their coboundaries, can be obtained by the integration of differential forms.

Here is one answer (Chapter IX, Theorem 5A) to this question, which impresses me as the most interesting theorem in the book. (This theorem was first proved in the 1948 Harvard Ph.D. thesis of the author's student J. H. Wolfe.) Consider all real valued $r$ dimensional cochains $X$ which are defined on the finite rectilinear simplicial chains in an open subset $R$ of Euclidean $n$-space, and for which there exists a real number $M$ such that if $\sigma$ is any $r$ dimensional simplex in $R$, then $|X(\sigma)|$ does not exceed $M$ times the $r$ dimensional measure of the point set spanned by $\sigma$; define the norm $|X|$ as the least such number $M$. If both $X$ and its $r+1$ dimensional coboundary $dX$ have finite norm, then $X$ and $dX$ can be computed by Lebesgue integration of bounded and measurable $r$ and $r+1$ dimensional differential forms, defined almost everywhere in $R$, and almost everywhere in each $r$ and $r+1$ dimensional simplex in $R$ respectively.

This theorem is a generalization of Rademacher's classical result asserting the differentiability almost everywhere of a function satisfying a Lipschitz condition. In fact, if $r=0$ and $R$ is convex, then $X$ is a real valued function, $|X|$ is the supremum of $X$, and $|dX|$ is the best Lipschitz constant for $X$. The proof of the present general theorem adds significant new features to the classical argument. From the finiteness of $|X|$ and $|dX|$ it follows that $X$ is alternating, and that $X$ is additive with respect to subdivision. Hence the classical theory of finitely additive set functions implies that in each $r$ dimensional plane the cochain $X$ is the indefinite integral of its bounded measurable derivative with respect to $r$ dimensional measure. The main difficulty is to show that this derivative depends continuously and even linearly, in the sense of Grassmann algebra, on the directions of all the $r$ dimensional planes through a point. The proof of continuity uses the finiteness of $|X|$ and $|dX|$ to show that $X$ has nearby values on the bottom simplex and the (not necessarily parallel) top simplex of a simplicial prism of small height. The proof of linearity uses the finiteness of $|dX|$ to verify the conditions of a known algebraic criterion for the multilinearity of a homogeneous alternating function.

Attaching to each cochain $X$ of the above type the new "flat" norm

$$|X|^{\flat} = \sup \{ |X|, |dX| \}$$