great importance in the study of transonic flow. The applications to this theory are given in several sections.

Two very useful features are the ample references to both quite recent and classical papers and the eighty or so representative problems gathered at the ends of the various sections.

The book is particularly to be recommended to anyone intending to work in the mathematical theory of hydrodynamics or aerodynamics.

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Infinite abelian groups. By I. Kaplansky. (University of Michigan Publications in Mathematics, no. 2.) Ann Arbor, University of Michigan Press, 1954. 5+91 pp. $2.00.

The theory of finite and infinite abelian groups is comparatively rich in structure theorems and these form the central theme of Kaplansky's excellent monograph. A satisfactory characterization is available, and presented here, for the groups in the following classes of abelian groups: finite groups [Frobenius-Stickelberger], torsion groups with bounded order, finitely generated groups, countable torsion groups [Ulm], groups with division, countably generated torsionfree modules over complete discrete valuation rings [Kaplansky]. Direct sums of cyclic groups are naturally prominent in this discussion, since the groups in the classes mentioned are either direct sums of cyclic groups or direct sums of cyclic groups and groups with division or else contain direct sums of cyclic groups as essential building blocks. Thus one finds here the theorems of Prüfer and Kulikoff, characterizing certain direct sums of cyclic groups, and their application proving that every subgroup of a direct sum of cyclic groups is itself a direct sum of cyclic groups.

Various structures may be derived from an abelian group. The ring of endomorphisms is known to reflect particularly faithfully the properties of the original group; and the author proves for a large class of groups that they are completely determined by their endomorphism rings—this is one of many instances where results in this work go beyond what had been known before. Similarly the theory of characteristic subgroups has been developed considerably further than had been done by the author's predecessors.

The first eleven chapters lead up to Ulm's theorem which is proved with admirable simplicity. This is no mean task, considering the difficulty of Ulm's original proof and of Zippin's simplified version thereof. Operators make their first appearance in the 12th chapter, quite rightly in our opinion, since in a large part of the theory of