RECENT DEVELOPMENTS IN THE THEORY OF LOCALLY CONVEX VECTOR SPACES

J. A. DIEUDONNÉ

1. **Introduction.** Although the theory of Banach spaces has been very popular among American mathematicians during the last twenty years, comparatively little attention seems to have been given, in this country, to its generalizations, except in the very last few years. With the exception of the outstanding work of G. W. Mackey [47; 48], most contributions to the general theory of locally convex spaces have been made by European mathematicians. There may be some interest, therefore, in a survey in broad outline of the most recent advances in that field, some of which have not yet appeared in print.

The principal motivation behind the general theory is the same as that of Banach himself: namely, a search for general tools which might be applied successfully to functional analysis. Two different sectors contributed the main influences: the first originated in the work of G. Köthe, O. Toeplitz, and their students on sequence spaces [32–46; 26; 50; 11], which began around 1934 and was partly related to the theory of functions of a complex variable [62]; many of the ideas which were to become fundamental in the later development of the general theory appeared there for the first time, and also a great wealth of illuminating examples and counter-examples. For unknown reasons, this remarkable pioneering work has to this day remained practically ignored in this country, in spite of its intrinsic importance and usefulness.

The other influence was exerted by the developments of the theory of integration, and chiefly through the efforts to free that theory from the shackles of the Carathéodory measure theory and turn it into a mere chapter of the general theory of topological vector spaces [6]. These efforts culminated in L. Schwartz's theory of distributions (1945), which could be expressed only in the language of locally convex vector spaces [56]; it turned out that for that theory, Banach spaces were an utterly inadequate tool, and the realization of that fact led to very active research on more general spaces, to which most of the results obtained in the last few years owe their origin.

This recent work has led in particular to a new classification and

An address delivered before the Chicago meeting of the Society on April 24, 1953 by invitation of the Committee to Select Hour Speakers for Western Sectional Meetings; received by the editors April 24, 1953.