PARAMETRIC SURFACES

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I want first to express my thanks to the Presidium of the American Mathematical Society for the invitation to deliver an address to the Society. I feel very much honored by this invitation and I appreciate it highly.

My address will be devoted to parametric surfaces, to that part of the subject which is connected with the definition of the area, with the expression of the area by an integral, and with the minimum area problem. It is an exceptional privilege to lecture on this subject to the American Mathematical Society, whose members have been responsible for such an impressive progress of the subject during the last two decades. I shall start with the definition of the area.

In 1914 Carathéodory defined the m-dimensional measure in the n-dimensional space, for integral values of m.[1] He considered the one-dimensional measure as a generalization of the length and he proved that the length of a rectifiable curve coincides with its one-dimensional measure.

In 1919, Hausdorff,[2] developing Carathéodory's ideas, constructed a continuous scale of measures. After the work of Carathéodory and Hausdorff the obvious problem was to treat the area as the two-dimensional measure and to establish the well known integral formulae. But a considerable amount of work on establishing the integral formulae for the area had been done before that, with a variety of definitions of the area, due no doubt to the difficulty of the problem. Later the Lebesgue definition, somewhat modified by Frechet, of the area as the lower limit of areas of approximating polyhedra became the dominant one. A particularly valuable feature of the Lebesgue-Frechet (L.-F.) definition is that it provides the area with the property of the lower semi-continuity.

When applied to plane figures bounded by a Jordan curve the L.-F. definition gives in fact the value of the interior area of the figure, and thus in cases when the boundary is of positive L_2-measure, the L.-F. area differs from the value of the closed area in the usual

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[1] *Über das lineare Mass von Punktmengen—eine Verallgemeinerung des Längenbegriffs*, Nachr. Ges. Wiss. Göttingen (1914) pp. 404–426.

[2] *Dimension und äusseres Mass*, Math. Ann. vol. 79 (1919) pp. 157–179.