AREA AND REPRESENTATION OF SURFACES

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What is surface and what is area? This question is not a trivial one, because study of the literature reveals that there are several relevant concepts of surface and area, just as there are several relevant concepts of curve and length (see [1, I.I, II.5, V.4]; numbers in square brackets refer to the bibliography at the end of this paper). These concepts are not only related to one another, but they are also related to the solution of many problems in which they play an essential part. My purpose is to discuss the connection of such concepts with calculus of variations and to show how the slow development of some of them is closely related to advances in calculus of variations.

During the war years I succeeded in developing a complete theory for Fréchet surfaces and Lebesgue area, which was related to previous work of McShane and Radó. It was with great pleasure that I learned after the war that, at the same time and independently, Radó and other mathematicians in this country had developed a theory founded upon the same fundamental ideas. The results obtained are partly overlapping and partly complementary. This allows us to combine our results into a single theory. Now this parallel development is not due to chance, but rather to an underlying common aim that has been a constant guide in our respective efforts: the aim to obtain a more general basis for the theory of double integrals in parametric form in calculus of variations. Indeed, the theory for such integrals has not yet been finally settled, as has been noticed by both Radó and Tonelli.

The concepts of curve and surface, length and area, were presented at other meetings of this society by J. W. T. Youngs [59] from a general point of view. Therefore I will limit myself here to showing only how much our concepts owe to calculus of variations, and then I will discuss the most recent and important results concerning the problem of area and representation of surfaces.

A complete list of all papers which are related to this topic would be too extensive for the purposes of the present paper. The bibliography at the end is not meant to be complete in any sense. The recent book of Radó, *Length and area* [1], will be used as a general refer-