TABLE OF THE ZEROS AND WEIGHT FACTORS OF THE FIRST FIFTEEN LAGUERRE POLYNOMIALS

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The chief use of the present table of zeros and weight factors of the Laguerre polynomials is in the performance of quadratures over the interval $[0, \infty]$, when the integrand behaves like the product of e^{-x} by a polynomial. From the well known theory of orthogonal polynomials, the quadrature formula is exact for any polynomial $P(x)$ up to the $(2n-1)$th degree, provided one uses the n-point quadrature formula. Thus if $\alpha_i^{(n)}$ denotes the "weight factors" or "Christoffel numbers," corresponding to $L_n(x)$, the nth Laguerre polynomial, and $x_i^{(n)}$ denotes the zeros of $L_n(x)$, then

$$\int_0^\infty e^{-x}P(x)dx = \sum_{i=1}^{n} \alpha_i^{(n)} P(x_i^{(n)}). \tag{1}$$

Besides problems involving direct quadratures, there are those arising in the numerical solution of linear integral equations, range $[0, \infty]$, where the unknown function occurs both inside and outside the integral sign. By considering the product of e^x by the integrand as a polynomial, and making use of (1), the approximation problem reduces to the solution of a set of only n linear equations. Hence only n points are needed to give accuracy obtainable by approximating the product of e^x by the integrand as a polynomial of the $(2n-1)$th degree. For a full description, including examples, see A. Reiz [3],[1] especially pp. 1–12.

For many purposes, the report of the Admiralty Computing Service [2], which furnishes zeros to 8 decimals and weight factors to 8 significant figures as far as $L_{10}(x)$, will suffice. This present table is intended to cope with problems requiring higher degree and accuracy. Thus there are given here the zeros and weight factors of the first fifteen Laguerre polynomials, the zeros to 12 decimals and the weight factors to 12 significant figures. (The zeros and weight factors are available in manuscript form to two extra places for $n \leqq 10$, and to one extra place for $10 < n \leqq 15$.) Also the example of A. Reiz is followed in that the quantities $\alpha_i^{(n)}e^{x_i^{(n)}}$ are also tabulated to 12 significant figures, to facilitate the quadratures when the integrand does not contain e^{-x} explicitly. Thus in

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[1] Numbers in brackets refer to the references cited at the end of the paper.