CORRECTION TO MY PAPER "NOTE ON AFFINELY CONNECTED MANIFOLDS"

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In my recent paper [1][1] *Note on affinely connected manifolds* I gave a proof of a theorem on affinely connected manifolds. As was pointed out by H. Whitney, the fact that I made use of in the proof, that the space of all real matrices (a_i^j), $|a_i^j|>0$, is simply connected, is erroneous. But the theorem I intended to prove is true. I present in the following lines a revised proof, in which are clarified at the same time certain ambiguities of that note.

We begin by considering the space $M(n)$ of all real matrices (a_i^j), $i, j=1, \cdots, n$ with $\Delta \equiv |a_i^j|>0$. Let $R(n)$ denote the group space of the proper orthogonal group in n variables. There is a natural way to imbed $R(n)$ in $M(n)$ and it is well known that $R(n)$ is a deformation retract of $M(n)$ [2]. In particular, it follows that $R(n)$ and $M(n)$ have the same homotopy type and hence isomorphic homotopy groups. Thus the fundamental group of $M(n)$ is free cyclic if $n=2$ and is cyclic of order two if $n\geqq 3$.

We denote by $\psi: R(n)\rightarrow M(n)$ the identity mapping and by $f: M(n) \rightarrow R(n)$ the deformation such that under f every point of $R(n)$ remains fixed. Let ψ and f denote at the same time the induced homomorphisms of the (singular) chains and ψ^* and f^* the corresponding dual homomorphisms of the cochains. Since f is a deformation, we have, for every one-dimensional cycle Z of $M(n)$, $Z\sim\psi f(Z)$. It follows that

$$\int_Z \frac{d\Delta}{\Delta} = \int_{\psi f(Z)} \frac{d\Delta}{\Delta} = \int_{f(Z)} \psi^*\left(\frac{d\Delta}{\Delta}\right) = 0,$$

since $\Delta=1$ in $R(n)$. In other words, in $M(n)$ the integral of $d\Delta/\Delta$ over any one-dimensional cycle is zero.

It is possible to express the differential form $d\Delta/\Delta$ in terms of a_i^j. In fact, let b_j^k be defined by $a_i^j b_j^k = b_i^j a_j^k = \delta_i^k$. Then it is easy to verify that

$$d\Delta/\Delta = da_i^j b_j^i.$$

These remarks on the group manifold being made, let us return to the affinely connected manifold M. Let $\mathfrak{F}$ be the vector bundle of all ordered sets of n linearly independent contravariant vectors through

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[1] Numbers in brackets refer to the references cited at the end of the paper.