A NOTE ON THE SINGULAR MANIFOLDS OF A DIFFERENCE POLYNOMIAL

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1. Introduction. In a previous paper[1] we defined essential singular manifolds of a difference polynomial in one unknown, and gave an example of such a manifold. By an obvious extension of this definition we may say that if A is an algebraically irreducible difference polynomial in unknowns $y_1, \cdots, y_n$, then an essential irreducible manifold of A which annuls a polynomial of lower effective[2] order than A in y_k, $1 \leqq k \leqq n$, or free of y_k is an *essential singular manifold of A relative to* y_k. The remaining essential irreducible manifolds of A we shall call, as in the case of a polynomial in one unknown, *ordinary manifolds relative to* y_k, and the totality of solutions in these manifolds the general solution *of A relative to* y_k,

The analogous situation in the theory of algebraic differential equations[3] suggests that the essential singular manifolds of a difference polynomial relative to one unknown are also essential singular manifolds relative to any other unknown. It is the purpose of this paper to show that this is actually the case. It will follow that we may drop the term "relative" from the concepts we have just defined. *The essential irreducible manifolds of an algebraically irreducible difference polynomial may be divided into two classes, singular manifolds and ordinary manifolds. The singular manifolds are, in the sense defined above, singular relative to each unknown present in the difference polynomial. The ordinary manifolds are ordinary relative to each unknown, and the totality of solutions they contain may be called the general solution of the difference polynomial.*

We make use, as in the theory of algebraic differential equations, of the *separants*[4] of a difference polynomial. Let A be a difference

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[1] *Manifolds of difference polynomials*, Trans. Amer. Math. Soc. vol. 64 (1948) pp. 133–172, referred to below as M.D.P.; §21.

[2] The effective order of a difference polynomial in y_k is defined in M.D.P. as the difference between the orders of the highest and lowest transforms of y_k appearing effectively in the polynomial.

[3] J. F. Ritt, *Differential equations from the algebraic standpoint*, Amer. Math. Soc. Colloquium Publications, vol. 14, 1932, p. 24.

[4] The perhaps unexpected fact that the separant plays a rôle in the theory of difference equations was observed by Poisson, *Mémoire sur les solutions particulières des équations differentielles et des équations aux différences*, J. École Polytech. vol. 6 (1806) pp. 60–125. Poisson's "particular solutions" do not necessarily lie in essential singular manifolds.