

BOOK REVIEWS

Foundations of algebraic geometry. By André Weil. (American Mathematical Society Colloquium Publications, vol. 29.) New York, American Mathematical Society, 1946. 20+288 pp. \$5.50.

In the words of the author the main purpose of this book is "to present a detailed and connected treatment of the properties of intersection multiplicities, which is to include all that is necessary and sufficient to legitimize the use made of these multiplicities in classical algebraic geometry, especially of the Italian school." There can be no doubt whatsoever that this purpose has been fully achieved by Weil. After a long and careful preparation (Chaps. I–IV) he develops in two central chapters (V and VI) an intersection theory which for completeness and generality leaves little to be desired. It goes far beyond the previous treatments of this foundational topic by Severi and van der Waerden and is presented with that absolute rigor to which we are becoming accustomed in algebraic geometry. In harmony with its title the book is entirely self-contained and the subject matter is developed *ab initio*.

It is a remarkable feature of the book that—with one exception (Chap. III)—no use is made of the higher methods of modern algebra. The author has made up his mind not to assume or use modern algebra "beyond the simplest facts about abstract fields and their extensions and the bare rudiments of the theory of ideals." Weil's faithful realization of this program of strict mathematical economy is an achievement in itself. In some cases this leads to the "best possible" proofs. However, on the whole one may question the wisdom of this self-imposed régime of austerity. The methodical reduction of the theory—which is both difficult and subtle—to the primitives of algebra is bound to be a very laborious process. As a result, the reader finds himself very much in the position of a man who must collect a large amount of cash most of which is in pennies. The author justifies his procedure by an argument of historical continuity, urging a return "to the palaces which are ours by birthright." But it is very unlikely that our predecessors will recognize in Weil's book their own familiar edifice, however improved and completed. If the traditional geometer were invited to choose between "makeshift constructions full of rings, ideals and valuations" on one hand, and constructions full of fields, linearly disjoint fields, regular extensions, independent extensions, generic specializations, finite specializations and specializations of specializations on the other, he most probably