BOOK REVIEWS

Eigenfunction expansions associated with second order differential equations. By E. C. Titchmarsh. Oxford, Clarendon Press, 1946. 8+174 pp. \$7.00.

The subject of this book has its origin in the Sturm-Liouville expansions. The author deals with the problem of expanding an arbitrary function in terms of the e.f's (eigenfunctions) of a second order ordinary differential equation, with emphasis on the singular theory. He puts aside H. Weyl's method of handling of the singular theory on the basis of integral equations and also bypasses the use of the general theory of linear operators in Hilbert spaces (M. H. Stone); instead, use is made of contour integrations and the Cauchy calculus of residues. The material in Chapters 1, 2, 4, 5, 7 is in some essential parts due to the author; Chapters 9, 10 are entirely due to him; the highly important Chapter 3 involves some very heavy technical equipment and is due essentially to the author and to H. Weyl; the material in Chapter 6 is due to M. H. Stone. The book is written with complete rigor in a very readable style. Specifically is studied the operator $L\equiv q(x)-d^2/dx^2$, where $q(x)$ is a given function, defined on some finite or infinite interval (a, b). The values of λ and the corresponding solutions (subject to suitable boundary conditions) of

$$Ly = \lambda y \tag{1}$$

are termed the e.v's (eigenvalues) and e.f's, respectively.

In the regular case (chap. 1) the following is proved. If real q is continuous on a finite interval (a, b), then (1) has a solution ϕ so that $\phi(a)=\sin\alpha$, $\phi'(a)=-\cos\alpha$ (α assigned); if, in addition, f is integrable over (a, b), then the Sturm-Liouville expansion $s(x)$ behaves with regard to convergence as an ordinary Fourier series, while $2^{-1}(f(x+0)+f(x-0))=s(x)$ when f is of bounded variation near x. In chap. 2 the author treats singular cases when the expansion is still a series and the interval is $(0, \infty)$. It is assumed that q is continuous on every finite subinterval of $(0, \infty)$. The general solution of (1) is of the form $\theta+l\phi$, where θ, ϕ are solutions of (1) such that $\theta(0)=-\phi'(0)=\cos\alpha$, $\theta'(0)=\phi(0)=\sin\alpha$. If solutions are considered for which $\{\theta(b)+l\phi(b)\}\cos\beta+\{\theta'(b)+l\phi'(b)\}\sin\beta=0$, one obtains $l=l(\lambda)=-[\theta(b)\operatorname{ctg}\beta+\theta'(b)][\phi(b)\operatorname{ctg}\beta+\phi'(b)]^{-1}$. As β varies, l describes a circle C_b; $C_b\to$a "limit-circle" or a "limit-point" as $b\to\infty$. For every non-real λ, (1) has a solution $\psi(x,\lambda)=\theta(x,\lambda)+m(\lambda)\phi(x,\lambda)$ $\subset L^2(0,\infty)$, where $m(\lambda)$ is the limit-point or is any point on the limit-