proved (i) to be necessary over any $E_m$-field by consideration of the structure of the full linear permutation group. Extending this method, all Galois groups of $x^m - a$ over $R$, possible under the conditions of the theorem, are established. (Received May 1, 1947.)

### 312. G. E. Wall: *Notes on binomic equations over an $E_m$-field.* II.

Let $x^m - a$ be irreducible over the $E_m$-field $R$, and reducible over $R(\epsilon)$ ($\epsilon$=primitive $m$th root of unity), and let $\alpha^m = a$. The form of the coefficient $a$ is determined so that $R(\alpha, \epsilon)$ has a given Galois group $\mathfrak{G}$ over $R$ (out of a class of admissible groups $\mathfrak{G}$ established in part I of this paper). Firstly, using the criteria of G. Darbi for normal binomic equations over an $E_m$-field (Annali di Matematica pura ed applicata (4) vol. 4 (1926)), the form of $a$ is determined so that $\mathfrak{G}$ is of given order. These results are then applied to certain binomic subfields of $R(\alpha, \epsilon)$ in order to distinguish between groups $\mathfrak{G}$ of equal order. (Received May 1, 1947.)

## Analysis

### 313. R. P. Boas: *Some complete sets of analytic functions.*

In the following theorems are generalized results of Ibragimov (Bull. Acad. Sci. URSS. Sér. Math. vol. 11 (1947) pp. 75–100). Let $f(z)$ be analytic in $|y| < \pi$ and of period $2\pi$. Let $\eta$ and $\zeta$ be positive numbers with $\eta < \pi$, $\zeta \leqq \pi - \eta$. Let a set of lower density $\zeta/\pi$ of the Fourier coefficients of positive index of $f(z)$ not vanish. Let $\alpha_n = \beta_n + i\gamma_n$ be a sequence of complex numbers, $0 \leqq \beta_n < 2\pi$. Then $\{f(z+\alpha_n)\}$ is complete in $|z| < \zeta$ in the following cases. (1) The set $\{\gamma_n\}$ has a limit point in $|y| < \eta$. (2) The function $f(z)$ is entire, of order $\rho$ and type $\sigma_1$, and $2\sigma_1 < \rho$ lim inf $n|\gamma_n|^{1-\rho}$. (Received July 22, 1947.)

### 314. R. P. Boas and K. Chandrasekharan: *Derivatives of infinite order.*

Let $f(x)$ have derivatives of all orders in $(a, b)$. The following theorems are proved. (1) If $f^{(n)}(x) \to g(x)$ for each $x$ in $(a, b)$, where $g(x)$ is finite, then $g(x) = Ae^x$. (2) If $f(x)$ belongs to a Denjoy-Carleman quasi-analytic class in the open interval $(a, b)$, and $\lim_{n\to\infty} f^{(n)}(x_0) = L$ exists for a single $x_0$ in $(a, b)$, then $f^{(n)}(x) \to Le^{x-x_0}$ in $(a, b)$. These theorems answer in the affirmative questions raised by V. Ganapathy Iyer (J. Indian Math. Soc. N.S. vol. 8 (1944) pp. 94–108) and remain true if $f^{(n)}(x) \to g(x)$ in a more general sense. If $\{\lambda_n\}$ is a given sequence of constants, the following theorems are proved. (3) Let (*) $\lim_{n\to\infty} f^{(n)}(x)/\lambda_n = g(x)$, $a \leqq x \leqq b$. If lim inf $|\lambda_{n-1}/\lambda_n| = 0$ and (*) holds uniformly, $g(x) \equiv 0$ in $(a, b)$. If lim inf $|\lambda_{n-1}/\lambda_n| > 0$ and (*) holds dominatedly, $g(x) = Ae^{Bx}$. (4) If lim sup $n^{-1}|\lambda_n|^{1/n} < \infty$ and (*) is true for each $x$ in $a < x < b$, then $g(x) = Ae^{Bx}$. (Received June 26, 1947.)

### 315. V. F. Cowling: *Some results for factorial series.*

Let $f(z) = \sum_{n=0}^{\infty} a_n n!/z(z+1) \cdots (z+n+1)$ with abscissa of convergence $\tau < \infty$. Let $l-1 < h < l$ where $l$ is integral and positive but otherwise arbitrary. Denote by $D$ the domain in the $w$-plane $\psi_2 \leqq \text{Arg } (w-h) \leqq \psi_1$ where $0 < \psi_1 \leqq \pi/2$ and $-\pi/2 \leqq \psi_2 < 0$. Let $a(w)$ be a function regular in $D$, with the possible exception of the point at infinity, for which $a(n) = a_n$, $n = 0, 1, \cdots$. Suppose for $w = h + Re^{i\psi}$ in $D$ and $R \geqq R_1$ that $|a(h+Re^{i\psi})| \leqq R^k \exp(-LR \sin \psi)$ where $k$ is arbitrary and $0 < L < 2\pi$. Then