THE ESSENTIAL PART OF A SURFACE

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This paper attempts to place the concept of the essential part of the projection of a continuous surface upon a plane in its historical and mathematical setting, and to outline the role this concept is playing in the solution of the area problem. To that purpose those events most closely related to this concept will be sketched, and less relevant facts will be suppressed.[1]

1. **The length of a curve.** Consider a continuous path curve C given by a representation

$$(1.1)\qquad C:\quad x = x(u),\ y = y(u),\ z = z(u), \qquad 0 \leqq u \leqq 1,$$

where the functions $x(u)$, $y(u)$, $z(u)$ are defined, single-valued, real-valued, and continuous on the closed unit interval $0 \leqq u \leqq 1$. It is well known that the length $L(C)$ of C may be defined as the limit of the lengths of inscribed polygons which converge to C—if $0 = u_0 < u_1 < \cdots < u_i < \cdots < u_n = 1$ be a subdivision of $0 \leqq u \leqq 1$, then

$$(1.2)\qquad \begin{aligned} L(C) = \lim_{n\to\infty} \sum_{i=1}^{n} \{ [x(u_i) - x(u_{i-1})]^2 &+ [y(u_i) - y(u_{i-1})]^2 \\ &+ [z(u_i) - z(u_{i-1})]^2 \}^{1/2}, \end{aligned}$$

the limit[2] being taken with respect to subdivisions of $0 \leqq u \leqq 1$ for which the maximum value of $|u_i - u_{i-1}|$ for i between 1 and n converges to zero with $1/n$. Observe that (1.2) gives an expression for the length of the curve C in terms of its representation (1.1), standard algebraic operations, and one limit process.

2. **The area of a surface.** Consider a continuous path surface S

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[1] A list of papers closely related to the facts to be presented in the sequel is included at the end of this paper. Numbers in brackets refer to this bibliography. For an exhaustive treatment of the concepts of length and area and an extensive bibliography upon the subject, the reader should consult Radó [4]. This volume of the Colloquium publications is now in the process of being published. The writer had the privilege of reading the manuscript.

[2] Of course, it is necessary to discuss the existence of this limit and its independence of the representation chosen for C. See Radó [4, III. 3].