PARAMETRIC SOLUTION OF LINEAR HOMOGENEOUS DIOPHANTINE EQUATIONS

WALLACE GIVENS

1. **Introduction.** Certain parametric expressions were recently shown by L. W. Griffiths [1][1] to yield all the integral solutions of a system of linear homogeneous equations with integer coefficients in terms of integral values of the parameters. A simpler proof of this same result is given in the next section (cf. our equations (4) with Griffiths' [1] equations (2)). Following this we prove that the denominator in the formulas may be made independent of the values assigned the parameters, a possibility not discussed by Miss Griffiths. The proof of this stronger theorem is made to depend on one of its special cases, namely a result of Hermite's which states that the n minors of order $n-1$ of an $n-1$ by n matrix can have preassigned integral values for a suitable choice of integral values of the components of the matrix.

2. **Form of the solution.** Without essential loss of generality, we may assume that the system of equations is linearly independent and contains more unknowns than equations. Let the equations be

$$\sum_{i=1}^{n} a_{\alpha i}x_i = 0, \qquad \alpha = 1, 2, \cdots, r \geqq 0, \tag{1}$$

where the $a_{\alpha i}$ are rational integers, the matrix $A = \|a_{\alpha i}\|$ is of rank r and we set $n = r+s+1$ with $s \geqq 0$. We shall suppose for convenience that the x_i have been numbered so that the determinant of $\|a_{\alpha\beta}\|$ (α, $\beta = 1, \cdots, r$) is different from 0.

We now adjoin to (1) the system of equations

$$\sum_{i=1}^{n} p_{\rho i}x_i = 0, \qquad \rho = 1, 2, \cdots, s, \tag{2}$$

and let D_i equal $(-1)^{i+1}$ times the determinant obtained by omitting the ith column of the $n-1$ by n matrix

$$\left\| \begin{matrix} A \\ P \end{matrix} \right\|, \qquad \text{where } P = \|p_{\rho i}\|. \tag{3}$$

Then $x_i = D_i$ is an integral solution of (1) and (2) for any choice of

Presented to the Society, December 28, 1946; received by the editors November 22, 1946, and, in revised form, January 11, 1947.

[1] Numbers in brackets refer to the references cited at the end of the paper.