A SUM CONNECTED WITH THE PARTITION FUNCTION

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1. **Introduction.** In the formula for the number $p(n)$ of unrestricted partitions of an integer n there appears the sum [1][1]

$$A_k(n) = \sum_{h \bmod k}{}' \omega_{h,k} \exp(-2\pi i h n/k), \tag{1}$$

where the dash ′ beside the summation symbol indicates here and in the sequel that the letter of summation runs only through a reduced residue system with respect to the modulus. The symbol $\omega_{h,k}$ denotes certain $24k$th roots of unity given by

$$\omega_{h,k} = \exp(\pi i s(h, k)),$$

where $s(h, k)$ is a Dedekind sum [2] defined by

$$s(h, k) = \sum_{\mu=1}^{k} \left(\left(\frac{\mu}{k}\right)\right)\left(\left(\frac{\mu h}{k}\right)\right).$$

The symbol $((x))$, in turn, is defined as follows:

$$((x)) = \begin{cases} x - [x] - 1/2 & \text{for } x \text{ not an integer,} \\ 0 & \text{for } x \text{ an integer,} \end{cases}$$

where $[x]$ denotes, as usual, the greatest integer not exceeding x. D. H. Lehmer [3] has investigated these sums on the basis of a different expression for the roots of unity involved. In the first place he factored the $A_k(n)$ according to the prime powers contained in k. Secondly, by reducing them to sums studied by H. D. Kloosterman [4] and H. Salié [5], he evaluated the $A_k(n)$ explicitly in the case in which k is a prime or a power of a prime. Both results together provide a method for calculating the $A_k(n)$. Alternate proofs of Lehmer's factorization theorems have been given in [2]. In the present note a new approach to the second of Lehmer's results is presented. The method given here is simpler than Lehmer's method, especially in the treatment of the case $k = 2^\lambda$.

2. **Some lemmas.** The proofs are based in part on three lemmas, which occur as Theorems 17, 18, 19 in [2].

LEMMA 1. *Let $\theta = \theta(k)$ denote 1 for $3 \nmid k$ and 3 for $3 \mid k$ so that θk is*

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[1] Numbers in brackets refer to the bibliography at the end of the paper.