certain functions not polynomials and for homogeneous polynomials of degree n in two real variables. (Received December 24, 1946.)

# 138. Bertram Yood: On ideals in operator rings over Banach spaces. Preliminary report.

Let R be the ring of all continuous linear operators on a Banach space X. For any ideal A in R let  $M \subset X$  be the set of all x in X for which T(x) = 0 for every T in A, and let N be the linear manifold in X generated by the ranges of T in A. It is shown that if A is a left (right) ideal in R then the closure of A in both the weak and strong topology of operators is the set of all T in R which vanish on all of M (set of all T in R whose range is contained in  $\overline{N}$ ). Thus for ideals the topologies are equivalent and the closed ideals characterized. These facts are used to show that if S is a subset of R, the left (right) annihilator of the right (left) annihilator of S in R is the smallest weakly closed left (right) ideal containing S. (Received January 6, 1947.)

### 139. Bertram Yood: Regular and singular elements in the ring of operators on a Banach space. Preliminary report.

The sets G and S of regular and singular elements respectively of the ring R of all bounded linear operators defined on an infinite-dimensional Banach space X are studied in the uniform, strong and weak topologies of R. In the uniform topology, S is decomposed into disjoint sets with topological properties in such a way that the properties of an operator T being an isomorphism or not, having X as its range or not, and the existence or absence of certain projections are correlated with the ring properties of being a left or right generalized null-divisor or not, and the possession or lack of a left or right inverse. If G can be dense in R then an open question of Banach (Théorie des opérations linéaires, p. 246) on linear dimensions has a negative answer. Banach's question is further discussed in terms of the concepts of this paper. In both the strong and weak topologies of R, it is shown that G and S are each dense and not open. (Received January 31, 1947.)

### 140. H. J. Zimmerberg: Definite integral systems.

In this paper the notions of definitely self-conjugate adjoint integral systems of Wilkins (Duke Math. J. vol. 11 (1944) pp. 155–166) and those definite systems of the author (Bull. Amer. Math. Soc. Abstract 52-3-76) are extended to integral systems written in matrix form  $y(x) = \lambda \int_a^b K(x, t) y(t) dt$ , where no restriction is made on the form of the kernel K(x, t). These definite integral systems include the definite systems previously treated as subclasses. With the exception of expansion theorems, most of the properties of the definite integral systems of Wilkins and the author are preserved. (Received January 8, 1947.)

#### APPLIED MATHEMATICS

# 141. J. B. Díaz and Alexander Weinstein: On an extremal property of the torsional rigidity.

The torsional rigidity S of a beam with simply or multiply connected cross section can be given by either of the following formulas, which have been hardly explicitly mentioned in the literature: (\*)  $S = P - D(\phi)$ , (\*\*)  $S = P - D(\psi)$ , where P is the polar moment of inertia,  $\phi$  and  $\psi$  are the warping function and its conjugate, and D denotes