GRATINGS AND HOMOLOGY THEORY

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1. Introduction. The relation of an abelian group to its character group is merely one example of a relation between an algebraic system Γ and a topological system X, each invariantively associated with the other. Of the two systems Γ and X, the first can be described in pseudo-combinatory terms, while the second involves the use of more sophisticated notions, such as passage to the limit, geometrical continuity, and so on. Accordingly, problems which are ordinarily expressed in terms of the system X can often be treated more simply by restating them in terms of the system Γ.

In this paper we shall be dealing with a class of algebraic systems Γ, called *gratings*. The theory of gratings will aim, among other things, to describe the topological properties of a space X in terms of the ways in which the space can be expressed as the union of two subspaces A and C, $A \cup C = X$. The theory will be pseudo-combinatory in character, in the sense that it will have to do with combinatory operations applied to an unlimited number of abstract elements, or symbols. It will acquire a geometrical significance only when the symbols are attached to appropriate geometrical entities. The theory will be applicable both to ordinary topological spaces and to spaces of a more general type, such as the ones determined by distributive lattices, which last need not be assumed to possess atomic elements. With the aid of gratings, we shall be able to reformulate a variety of problems in topology, differential geometry, potential theory, and so on, in pseudo-combinatory terms.

The present paper will be devoted almost exclusively to the elementary formal theory of gratings. Further developments, along with a number of typical applications, will be considered subsequently.[1]

2. Cuts, elements, gratings, cells. A *cut* will be any ordered triad $\gamma = (a, b, c)$ consisting of three different *abstract elements* a, b, and c. The element a will be called the *negative face*, the element b the *edge*, the element c the *positive face* of the cut.

A *grating* $\Gamma = [\gamma]$ will be any (finite or infinite) set of cuts, such that no two of the cuts have an element in common. The cuts γ will be

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[1] The terminology adopted in this paper differs quite radically from that used by the author in an earlier paper on gratings: *A theory of connectivity in terms of gratings*, Ann. of Math. vol. 39 (1938) pp. 883–912.