THEOREM 3. *If the conditions of Theorem 2 part A are satisfied and if in addition the quantities ψ_1, ψ_2 and δ satisfy the inequality*

$$2\pi > \delta(\csc \psi_1 - \csc \psi_2)$$

then the circle of convergence is not a cut for the function.

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A NOTE ON THE HILBERT TRANSFORM

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The Hilbert transform of $f(t)$, $-\infty < t < \infty$, is $1/\pi$ times the Cauchy principal value

$$\bar{f}(x) = P\int_{-\infty}^{\infty} \frac{f(t)}{t-x}\,dt = \lim_{\delta\to 0+}\int_{\delta}^{\infty} \frac{f(x+t) - f(x-t)}{t}\,dt.$$

If $f(t) \in L_p$, $p > 1$, then $\bar{f}(x) \in L_p$, and a considerable literature is devoted to studying the relationship of such pairs of "conjugate" functions to the theory of functions analytic in a half-plane. More to the point of the present note is a series of papers studying the Hilbert transform along strictly real variable lines ([2, 3]; further bibliography in [2]).[1]

Much less is known about $\bar{f}(x)$ when $f(t) \in L_1$. Plessner found by applying complex variable methods to the theory of Fourier series that if $f(t) \in L_1$ then $\bar{f}(x)$ exists almost everywhere (see [1, p. 145]). Besicovitch [4] proved Plessner's result using only the theory of sets, starting from his own previous real variable investigation of the L_2 transform case. S. Pollard [5] showed how Besicovitch's proof could be extended to prove the existence a.e. of the principal value of the Stieltjes integral

$$\bar{f}(x) = P\int_{-\infty}^{\infty} \frac{dF(t)}{t-x},$$

Received by the editors April 11, 1946.

[1] Numbers in brackets refer to the bibliography at the end of the paper.