ON A PROBLEM OF A. KUROSCH

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The following problem was posed by Kurosch [2]:[1] Suppose that an algebraic algebra[2] A over a field F has a finite number of generators over F, and suppose further that A is of bounded degree over F. Is then A of finite dimensionality over F? Kurosch answered his problem in the affirmative in the case where the elements of the algebra are of degree not greater than 3. As was observed by N. Jacobson [1], Kurosch's problem is equivalent to the following: Is any algebraic algebra of bounded degree locally finite[3] over F? Jacobson succeeded in reducing this problem to the special case of nil-algebras. In the present note we supplement Jacobson's results by proving that *any nil-algebra of bounded index (degree) is locally finite.* Jacobson's results combined with this theorem answer in the affirmative the question raised by Kurosch.

To obtain this result we make use of the notion of semi-nilpotency[4] and of its counterpart, the semi-regularity, which were introduced by the author in a previous paper [3]. As may be easily verified, each finitely generated nilpotent algebra is of finite dimensionality, and conversely, each nil-algebra of finite dimensionality is nilpotent. Hence, for nil-algebras the notion of local finiteness coincides with the notion of semi-nilpotency. Thus the above formulated result concerning nil-algebras is a consequence of the following more general theorem concerning nil-rings which will be proved in this paper: *Each nil-ring of bounded index is semi-nilpotent.* It is of interest to note in this connection that an analogous theorem does not hold for associative multiplicative systems (semi-groups).[5]

It will be convenient to adopt the terminology introduced by the author in [3]. Thus we use here the term radical to denote the sum $N(S)$ of all two-sided semi-nilpotent ideals of a ring S. We shall need the following theorems which were proved in [3]:

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[1] Numbers in brackets refer to the bibliography at the end of the paper.

[2] For the terminology compare Jacobson [1].

[3] An algebra A is called locally finite over F if each finite set of elements of A generates an algebra of finite dimensionality over F.

[4] A ring is called semi-nilpotent if each finite set of elements in this ring generates a nilpotent ring. A ring which is not semi-nilpotent is called semi-regular.

[5] Compare Kurosch [2, proof of Theorem 5]. This was observed independently also by Dr. Th. Motzkin of Jerusalem.