RECURSIVE PROPERTIES OF TRANSFORMATION GROUPS

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Introduction. If a continuous flow, the transformations of which are denoted by f^t, is defined in a topological space X, a point x of the space can be classified according to the behavior of the continuous orbit of the point. Among the types of points which have been considered are the periodic points, almost periodic points and recurrent points. If we fix the value of the parameter t, we obtain a transformation of the space X into itself. This transformation and its iterates determine a "discrete" flow and the "discrete" orbit of a point. Again a point of the space can be classified according to the behavior of the discrete orbit of the point. The question then arises as to whether a point belongs to the same class in the two cases.

Since the continuous orbit of a point contains the discrete orbit, many properties are retained when we pass from the discrete flow to the continuous flow. For example, if a point is periodic with respect to the discrete flow, it is clearly periodic with respect to the continuous flow. It is in the passage from the continuous to the discrete flow that the results are more difficult to predict.

In this note, the problem is generalized by replacing the parameter space of reals by an additive, abelian, locally compact, topological group. The action of such a transformation group T on a point and the action of certain subgroups G on the point are then related. It is shown that a general recursive property of T carries over to G (Theorem 1). It follows immediately that in the case of a continuous flow, if a point is either almost periodic, recurrent, or strongly recurrent with respect to the continuous flow, it retains this property with respect to the discrete flow determined by fixing the parameter t. It also follows that these properties carry over from a discrete flow to a sub-discrete flow. (For recurrence and almost periodicity, this was first proved by Gottschalk [2][1] and subsequently extended to discrete semi-flows by Erdös and Stone [1].)

Theorem on recursive points. Let X be a topological space and let T be an additive abelian locally compact topological group. Let f be a continuous transformation of $X \times T$ into X. We agree to write $f^t(x)$ in place of $f(x, t)$ $(x \in X, t \in T)$ whenever we wish. Furthermore, let f

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[1] Numbers in brackets refer to the Bibliography at the end of the paper.