Geometry

181. Reinhold Baer: *Projectivities with fixed points on every line of the plane.*

The class of projectivities under consideration comprises perspectivities and involutions. These two types exhaust our class in case the Theorem of Pappus holds. But already in the projective plane with coordinates from the field of real quaternions there may be found projectivities which are neither perspectivities nor involutions, and which still belong to our class. The properties of this class are found to be particularly striking, in case the projective plane under consideration is finite. (Received March 25, 1946.)

182. H. S. M. Coxeter: *The content of the general regular polytope.*

The regular solid $\{p, q\}$ has q p-gons at each vertex; that is, it has face $\{p\}$ and vertex figure $\{q\}$. Analogously, the four-dimensional polytope $\{p, q, r\}$ has cell $\{p, q\}$ and vertex figure $\{q, r\}$; and the n-dimensional polytope $\{p, q, \cdots, v, w\}$ has cell $\{p, q, \cdots, v\}$ and vertex figure $\{q, \cdots, v, w\}$. It is found that such a polytope of edge $2l$ has content $(g/n!)l^n b_1^{n/2}/b_1 b_2 \cdots b_{n-1} b_n^{1/2}$, wherein g is the order of the symmetry group and b_k the denominator of the kth convergent of the continued fraction $1/c_1-1/c_2-1/c_3-\cdots$, $c_1c_2=\sec^2 \pi/p$, $c_2c_3=\sec^2 \pi/q$, $\cdots$, $c_{n-1}c_n=\sec^2 \pi/w$. In the case of the n-dimensional hypercube $\{4, 3, \cdots, 3, 3\}$, $g=2^n n!$. Write $c_1=1$, $c_2=c_3=\cdots=2$, whence every $b_k=1$. (The kth convergent is simply $k/1$.) (Received March 6, 1946.)

183. John DeCicco: *Geodesic perspectivities upon a sphere.*

The following fundamental theorem is proved. If, under a perspective map of a surface Σ upon a sphere S, more than $3\infty^1$ geodesics of Σ correspond to the great circles of S, then every geodesic of Σ is mapped into a great circle of S, and Σ is a sphere or a plane. If Σ is a plane, the author obtains gnomic projection. Otherwise if Σ is a sphere, then both S and Σ are homothetic with respect to the point of perspectivity O and the corresponding points on S and Σ must be homothetic with respect to O. This may be contrasted with the corresponding result obtained by Kasner and the author, on conformal perspectivities of a sphere, where a point of S can correspond to either or both of the two perspective images on the sphere Σ. (Received March 7, 1946.)

184. Arnold Emch: *Dissection of two equivoluminal parallelotops into two finite series of equal numbers of congruent pieces in ordinary and higher Euclidean spaces.*

For the ordinary plane E_2 this problem has been known for a long time. Dehn has proved that in general it is not possible in E_3. By a new method based upon a particular affine relation between two equiareal rectangles it is shown how two equiareal polygons in E_2 can be dissected in the manner indicated above. In E_3 the designation "box" (Kiste by Schouten) is taken as a synonym for "rectangular parallelopipedon." The same term shall be used for rectangular parallelotop in $E_4, \cdots, E_n$, where the n edges on a vertex are mutually perpendicular. Based on an extension of the method for E_2 it is then successively shown how the problem can be solved for $E_3, E_4, \cdots, E_n$. The preliminary dissection of a parallelotop into an equivoluminal box in any E_n is not difficult. (Received February 19, 1946.)