1932, pp. 408, 409) and L. Bers and A. Gelbart (Quarterly of Applied Mathematics vol. 1 (1943) pp. 168–188; Bull. Amer. Math. Soc. vol. 50 (1944) p. 56). For all real  $p \ge 1$ ,  $\phi(x, y)$  is interpreted as the potential of a (p+2)-dimensional axially symmetric flow with a stream function  $\psi(x, y)$  given by  $\psi_x = -y^p \phi_y, \psi_y = y^p \phi_x$ . It is shown that the fundamental solution  $\phi_0(x, y)$  of (\*), with a singularity at  $(x_0, y_0)$ , can be expressed in terms of Bessel functions, and that the corresponding  $\psi_0(x, y)$  is many-valued, except for  $y_0=0$ . An application of the divergence theorem yields the period of  $\psi_0$  and leads to the determination of the various limits of the integral  $\int_0^\infty \exp(-|x|s) J_q(as) J_{q+1}(ys) ds$  for  $x \rightarrow 0$  and  $y \rightarrow b$ , where q denotes a non-negative real number. Corresponding results can be formulated in terms of generalized Laplace integrals. (Received February 28,

1946.)

## 172. J. E. Wilkins: A note on the general summability of functions.

There is a result of Titchmarsh (Introduction to the theory of Fourier integrals, Oxford, 1937) which gives sufficient conditions for the relation  $\int K(x, y, \delta)f(y)dy = f(x) + o(1)$ as  $\delta \to 0$ . In case  $K(x, y, \delta)$  has the form  $\{\Gamma(\lambda+1)/2\pi^{1/2}\Gamma(\lambda+1/2)\}\cos^{2\lambda}(1/2)(y-x)$ , where  $\lambda = 1/\delta$ , then Natanson (On some estimations connected with singular integral of C. de la Vallée Poussin, C. R. (Doklady) Acad. Sci. URSS. vol. 45 (1944) pp. 274– 277) has shown that the remainder term o(1) may be written as  $\delta f''(x) + o(\delta)$ . The author proposes to provide an extensive generalization of Titchmarsh's theorem to give sufficient conditions for the existence of an asymptotic expansion for the integral of  $K(x, y, \delta)f(y)$ . The author will also apply the general theory thus developed to several interesting kernels  $K(x, y, \delta)$ , and in particular will obtain the asymptotic expansion of which Natanson gave the first two terms. (Received March 21, 1946.)

173. H. J. Zimmerberg: Two general classes of definite integral systems.

In this paper the notions of definite integral systems considered by Reid (Trans. Amer. Math. Soc. vol. 33 (1931) pp. 475–485), Wilkins (Duke Math. J. vol. 11 (1944) pp. 155–166) and the author (Bull. Amer. Math. Soc. Abstract 52-3-77) are extended to integral systems written in matrix form  $y(x) = \lambda \{A(x)y(a) + B(x)y(b) + \int_a^b K(x, t)y(t)dt\}$ , where the  $n \times n$ -matrix  $K(x, t) \equiv H(x, t)S(t)$  and the  $n \times 2n$  matrix  $||A(x)B(x)|| \equiv ||H(x, a)H(x, b)|| G$ , G denoting a  $2n \times 2n$  constant matrix. These integral systems include the system of integral equations to which a system of first-order linear definite differential equations containing the characteristic parameter linearly in the two-point boundary conditions is equivalent. It is also shown that an integral system of the above form is equivalent to a system of 3n homogeneous equations of Fredholm type. (Received February 27, 1946.)

## APPLIED MATHEMATICS

174. Nathaniel Coburn: Pressure-volume relations and the geometry of the net of characteristics in two-dimensional supersonic flows.

In a previous paper (Quarterly of Applied Mathematics vol. 3 (1945) pp. 106–116), the author showed that for the Karman-Tsien pressure-volume relation in the twodimensional supersonic flow of a perfect, irrotational, compressible fluid, the net of characteristics (Mach lines) consists of a Tschebyscheff (fish) net. In the present paper,