229. C. A. Truesdell: On the functional equation  $\partial F(z, \alpha)/\partial z = F(z, \alpha+1)$ .

It is attempted to provide a theory which motivates and verifies many seemingly special relations among various familiar special functions. The recurrence relation  $\partial F(z, \alpha)/\partial z = A(z, \alpha)F(z, \alpha) + B(z, \alpha)F(z, \alpha+1)$  satisfied by many familiar functions furnishes common ground for study. In all cases of interest this equation is reducible to the form  $\partial F(z, \alpha)/\partial z = F(z, \alpha+1)$ . If  $\phi(\alpha)$  is bounded in a right half-plane a unique solution  $F(z, \alpha)$ , an integral function of z, exists such that  $F(z_0, \alpha) = \phi(\alpha)$ . Two solutions which agree in a right half-plane of  $\alpha$  when  $z = z_0$  agree for all values of z whether or not they are bounded functions of  $\alpha$ . On the basis of these theorems it is possible to establish many relationships satisfied by certain classes of solutions of the F-equation: (1) power series solutions, (2) factorial and Newton series solutions, (3) contour integral solutions, (4) generating expansions, (5) definite integrals, (6) relations among various different solutions of the F-equation. Methods of discovery are stressed because the discovery of a relationship satisfied by some special function or functions is almost always more difficult and more interesting than the construction of an ad hoc rigorous proof. A number of these special relations are shown to be obtainable by substitution in general formulas. (Received August 7, 1945.)

## APPLIED MATHEMATICS

## 230. R. J. Duffin: Nonlinear networks. I.

A system of n nonlinear algebraic equations in n real variables is studied and shown to have a unique solution. A special case of this system is the equations which govern the distribution of current in a direct current electrical network when the conductors are *quasi-linear*. A quasi-linear conductor is one in which the potential drop across the conductor and the current through the conductor are nondecreasing functions of one another. It follows that the distribution of current among the conductors of a quasi-linear network is unique. (Received September 6, 1945.)

## 231. F. J. Murray: Linear equation solvers.

The theory and actual construction of certain devices for the solution of a system of linear equations  $\sum_{j=1}^{n} a_{i,j}x_j = b_i$  is described. In these machines, the variables  $x_j$  are subject to the control of the operator and the machine indicates the value of  $\mu = \sum_{i=1}^{n} (\sum_{j=1}^{n} a_{i,j}x_j - b_i)^2$ . The operator varies each  $x_j$  in turn to minimize this expression. This is equivalent to the Gauss-Seidel method applied to the symmetric system obtained by multiplying the given set of equations by the adjoint matrix. The expressions  $\sum_{j=1}^{n} a_{i,j}x_j - b_i$  are realized (except possibly in sign) as the amplitudes of alternating current voltages by means of a combination of bell transformers and variable resistances. This can be done in a number of ways. These alternating currents are then rectified by means of diode vacuum tubes and the combined currents measured by a microammeter. The result is essentially  $\mu$ . Emphasis is placed upon the possibility of amateur construction and standard radio parts are used. The cost of the part of the device associated with the coefficients is proportional to  $n^2$ . The sensitivity of modern vacuum tubes is utilized to minimize the constant of proportionality. Received August 3, 1945.)

232. H. E. Salzer: Note on coefficients for numerical integration with differences.