A NOTE ON SYSTEMS OF HOMOGENEOUS ALGEBRAIC EQUATIONS

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1. **Introduction.** Consider a system of algebraic equations

$$
\begin{aligned}
f_1(x_1, x_2, \cdots, x_n) &= 0,\\
f_2(x_1, x_2, \cdots, x_n) &= 0,\\
\cdots\cdots&\cdots\\
f_h(x_1, x_2, \cdots, x_n) &= 0,
\end{aligned}
\tag{1}
$$

where f_i is a homogeneous polynomial of degree r_i with coefficients belonging to a given field K. We interpret $x_1, x_2, \cdots, x_n$ as homogeneous coordinates in an $(n-1)$-dimensional projective space. When $n>h$, the system (1) has non-trivial solutions $(x_1, x_2, \cdots, x_n)$ in an algebraically closed extension field of K, but there may not exist any such solutions in K itself. It is, in general, extremely difficult to decide whether adjunction of irrationalities of a certain type to K is sufficient to guarantee the existence of non-trivial solutions of (1) in the extended field. However, the situation is much simpler, when n is very large, in the sense that n lies above a certain expression depending on the number of equations h and the degrees $r_1, r_2, \cdots, r_h$.

We shall show:

THEOREM A. *For any system of h positive degrees $r_1, r_2, \cdots, r_h$ there exists an integer $\Phi(r_1, r_2, \cdots, r_h)$ such that for $n \geqq \Phi(r_1, r_2, \cdots, r_h)$ the system* (1) *has a non-trivial solution in a soluble extension field K_1 of K. The field K_1 may be chosen such that its degree N_1 over K lies below a value depending on $r_1, r_2, \cdots, r_h$ alone and that any prime factor of N_1 is at most equal to* $\max(r_1, r_2, \cdots, r_h)$.

This Theorem A is evidently contained in the following theorem.

THEOREM B. *For any system of positive integers $r_1, r_2, \cdots, r_h$ and any integer $m \geqq 0$, there exists an integer $\Phi(r_1, r_2, \cdots, r_h; m)$ with the following property: For $n \geqq \Phi(r_1, \cdots, r_h; m)$, there exists a soluble extension field K_2 of K such that all points $(x_1, x_2, \cdots, x_n)$ of an m-dimensional linear manifold L, defined in K_2, satisfy the equations* (1). *Here K_2 may be chosen so that its degree N_2 over K lies below a bound depending on $r_1, r_2, \cdots, r_h$ and m and that no prime factor of N_2 exceeds* $\max(r_1, r_2, \cdots, r_h)$.

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