A CORRECTION TO MY BOOK ON TOPOLOGICAL GROUPS

ANDRÉ WEIL[1]

On p. 28 of my monograph on topological groups,[2] implicit use is made of the following proposition: "If G is a topological group, and g a closed invariant subgroup of G, then the factor-group G/g cannot have a dimension greater than that of G." This is a very plausible but hitherto unproved result; the assertions on the dimension of a projective limit (IGTA, p. 28) must therefore partake of its conjectural nature. However, the results on the dimension of compact groups and of locally compact abelian groups (IGTA, chap. 5, §25, and chap. 6, §29) can be justified by dealing with a special case of that proposition; to do so is the purpose of the present note. Since no satisfactory theory of dimension exists except for "separable metrizable" spaces,[3] it will be enough to consider the case of groups which are metrizable and separable. A compact group which satisfies these conditions will be called *enumerably compact* in this note; such a group has at most enumerably many irreducible linear representations; it follows from this, and from the results of IGTA, §25, that it is the projective limit of a *sequence* of compact Lie groups. The result to be proved is now as follows.

THEOREM. *Let G be an enumerably compact group. Let g_0 be a closed invariant subgroup of G such that G/g_0 is a Lie group. Let U_0 be a neighborhood of the unit-element in G/g_0, homeomorphic to the interior of a sphere; and let X be the set of all elements of G, the image of which in G/g_0 is in U_0. Then X is homeomorphic to the (topological) direct product $U_0 \times g_0$.*

Let ϕ be the canonical mapping of G on the factor-group $G_0 = G/g_0$, i.e., the mapping in which, to every element of G, corresponds its co-set with respect to g_0. As is well known, our theorem will be proved if we show that we can find in G a system of representatives U for U_0, i.e., a homeomorphic image $U = \psi(U_0)$ of U_0 in G such that the inverse ψ^{-1} of ψ coincides on U with ϕ. In order to do that, we write G