Segre. Consider the two quadrics of Moutard belonging to the tangents t and t' at a point of the surface; they intersect in the asymptotic tangents and a conic. The limiting position of the plane of this conic as $t' \to t$ is

$$\begin{aligned} z[4\beta^2 - 4\gamma^2 n^6 - 2\beta n\phi + 2\gamma n^5\psi + 4\beta\psi n^2 - 4\gamma\phi n^4] \\ + 8(\gamma n^3 + \beta)n^2 x - 8(\beta + \gamma n^3) n y = 0, \end{aligned} \tag{15}$$

which envelopes the cone of Segre.

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ON A REPRESENTATION IN SPACE OF GROUPS OF CIRCLE AND TURBINE TRANSFORMATIONS IN THE PLANE

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1. Introduction. In a previous paper [6][1] the author showed that the oriented lineal elements in the euclidean plane can be mapped continuously and (1, 1) upon the points of *quasi-elliptic*[2] three-space Q_3 so that the whirl-similitude group of turbine[3] transformations in the euclidean plane is represented isomorphically upon the group of projective automorphisms of Q_3. By means of this representation *proper* turbines in the plane are mapped upon those real lines in Q_3 which do not intersect a line L, the real part of the quasi-elliptic absolute. It is the purpose of this note to investigate this representation analytically and to extend it so as to yield a (1, 1) continuous mapping of the turbines (proper and improper) in the *Moebius* plane upon all the lines in projective S_3. By such means we establish the isomorphism between certain groups of projective transformations in space on the one hand, and on the other of Kasner's 15-parameter group of turbine transformations [7] and some of its important subgroups, namely the Moebius, Laguerre, and Lie groups of circle transformations.

Other representations of turbines in space are due to Kasner and DeCicco [7, 8] and to A. Narasinga Rao [9]. The former use a

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[1] The numbers in brackets refer to the bibliography at the end of this paper.

[2] The term *quasi-elliptic space* is due to Blaschke [1, 2]. The absolute of this space is composed of a pair of conjugate imaginary planes $x_3^2+x_4^2=0$ and a pair of conjugate imaginary points $(1:\pm i:0:0)$.

[3] The geometry of turbines was initiated by Kasner [7]. An extensive bibliography on the subject is to be found in [5].