# ON THE QUADRICS ASSOCIATED WITH A POINT OF A SURFACE

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It is well known that among the three-parameter family of quadrics having a contact of the second order with an analytic surface at a point there are two systems of quadrics known as the quadrics of Darboux and the quadrics of Moutard. The particular quadric of the first family introduced in this note is of some interest. We propose to give a new geometrical construction for the cone of Segre by using Moutard quadrics and study the canonical pencil of lines associated with the surface.

Let $x^i(u, v)$ $(i=1, \cdots, 4)$ be the projective normal coordinates of a point on a surface $S$ referred to its asymptotic net $(u, v)$; then the functions $x$ are solutions of a completely integrable system of differential equations in Fubini's canonical form

$$\begin{aligned} x_{uu} &= \theta_u x_u + \beta x_v + p_{11} x, \\ x_{vv} &= \gamma x_u + \theta_v x_v + p_{22} x, \\ \theta &= \log \beta\gamma. \end{aligned} \tag{1}$$

As usual we introduce nonhomogeneous local coordinates of a point with respect to the tetrahedron $(xx_u x_v x_{uv})$, so that the equation of any quadric of Darboux is found to be

$$z - xy + kz^2 = 0, \tag{2}$$

where $k$ denotes a parameter, while the equation of the quadric of Moutard which belongs to the tangent $y-nx=0$, $z=0$ is

$$\begin{aligned} &[4(\beta + \gamma n^3)^2 + 3n(\beta_u + 4\beta_v n + 4\gamma_u n^3 + \gamma_v n^4)]z^2 \\ &\quad + 36n^3\left[z - xy + \frac{z^2}{2}\frac{\partial^2 \log \beta\gamma}{\partial u \partial v}\right] \\ &\quad - 12n(\beta - 2\gamma n^3)\left(y + \frac{1}{2}\frac{\partial \log \beta\gamma}{\partial u} z\right)z \\ &\quad - 12n^2(\gamma n^3 - 2\beta)\left(x + \frac{1}{2}\frac{\partial \log \beta\gamma}{\partial v} z\right)z = 0. \end{aligned} \tag{3}$$

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