For by Proposition 4, $FA = A$ for every A and $A(A^{-1}F) = EF = F$.

Let $S_1, \cdots, S_n$ be n statements. Let A_i be the statement: "All the preceding statements are annulled but S_i is true." It is interesting to note that the statements A_i form an idempotent (l, r) system.

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A CONJECTURE IN ELEMENTARY NUMBER THEORY

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A well known conjecture of Catalan states that *if $f(n)$ is the sum of all divisors of n except n, then the sequence of iterates of $f(n)$ is either eventually periodic or ends at* 1. It not only seems impossible to prove this, but it is also very difficult to verify.[1]

Another conjecture of Poulet,[2] which appears equally difficult to prove, has the doubtful merit that it is easy to verify. Let $\sigma(n)$ be the sum of all divisors of n, and let $\phi(n)$ be Euler's function. Then *for any integer n the sequence*

$$f_0(n) = n, \qquad f_{2k+1}(n) = \sigma(f_{2k}(n)), \qquad f_{2k}(n) = \phi(f_{2k-1}(n))$$

is eventually periodic.

We have verified this conjecture to $n = 10000$ (extending Poulet's verification) by using Glaisher's tables.[3] The checking was facilitated by the following observation: if the conjecture is to be checked for all $n < x$, it is enough to find a member of the sequence other than the first which is less than x.

The longest cycle found was in the sequence $f_k(9216)$. It starts with $f_6(9216)$, and is: 34560, 122640, 27648, 81800, 30976, 67963, 54432, 183456, 48384, 163520, 55296, 163800, 34560. However our method of checking does not show that this is the largest cycle up to 10000, and in fact Poulet found that $f_k(1800)$ has the same length 12.

As a rule $\phi(\sigma(n))$ is less than n. In fact, it can be shown that *for every $\epsilon > 0$, $\phi(\sigma(n)) < \epsilon n$, except for a set of density* 0. The proof follows from the following two observations:

Received by the editors May 18, 1944.

[1] L. E. Dickson, *Theorems and tables on the sum of the divisors of a number*, Quart. J. Math. vol. 44 (1913) pp. 264–296, and P. Poulet, *La Chasse aux Nombres*, vol. 1, pp. 68–72, and vol. 2, p. 188.

[2] P. Poulet, *Nouvelles suites arithmétiques*, Sphinx vol. 2 (1932) pp. 53–54.

[3] J. W. L. Glaisher, *Number-divisor tables*, British Association for the Advancement of Science, Mathematical Tables, vol. 8.