THE COMPLETION OF A THEOREM OF KANTOR

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The nature of the problem. Let a homaloidal net be defined by its order n and its multiplicities $s_1, s_2, \cdots, s_\rho$ at set P_ρ^2 of ρ general points in the plane. The positive integers $n, s_1, s_2, \cdots, s_\rho$ satisfy the equations

$$(1)\qquad \begin{aligned} s_1^2 + s_2^2 + \cdots + s_\rho^2 - n^2 &= -1, \\ s_1 + s_2 + \cdots + s_\rho - 3n &= -3. \end{aligned}$$

A planar Cremona transformation C is set up by putting this net into projective correspondence with a net of lines in another plane. If a complete and regular linear system $\Sigma_{p,d}$ of dimension d, the generic curve of the system having the genus p, has the order x_0 and the multiplicities $x_1, x_2, \cdots, x_\rho$, then $x = \{x_0; x_1, \cdots, x_\rho\}$ is called the characteristic of $\Sigma_{p,d}$. The image of $\Sigma_{p,d}$ under C is another linear system of the same p, d and a characteristic $x' = \{x_0'; x_1', \cdots, x_\rho'\}$ at the set Q_ρ^2 of the fundamental points of C^{-1}. x' is related to x by the substitution

$$(2)\qquad \begin{aligned} x_0' &= nx_0 - r_1x_1 - \cdots - r_\rho x_\rho, \\ x_1' &= s_1x_0 - \alpha_{11}x_1 - \cdots - \alpha_{1\rho}x_\rho, \\ &\cdots\cdots\cdots\cdots\cdots\cdots, \\ x_\rho' &= s_\rho x_0 - \alpha_{\rho 1}x_1 - \cdots - \alpha_{\rho\rho}x_\rho. \end{aligned}$$

The sets of numbers $\{s_i, \alpha_{i1}, \cdots, \alpha_{i\rho}\}$ are the characteristics of the principal curves of C at P_ρ^2 and satisfy the equations

$$(3)\qquad \begin{aligned} \alpha_{i1}^2 + \alpha_{i2}^2 + \cdots + \alpha_{i\rho}^2 - s_i^2 &= 1, \\ \alpha_{i1} + \alpha_{i2} + \cdots + \alpha_{i\rho} - 3s_i &= -1. \end{aligned}$$

The linear substitution (2) has as absolute invariants the forms

$$(4)\qquad \begin{aligned} (xx) &\equiv x_1^2 + x_2^2 + \cdots + x_\rho^2 - x_0^2; \\ (lx) &\equiv x_1 + x_2 + \cdots + x_\rho - 3x_0. \end{aligned}$$

The problems considered in this paper arise from the fact that the converses of two of the above statements do not hold. There are sets of positive integers satisfying equations (1) which are not associated with any homaloidal net and there are linear substitutions (2) which

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