ANALYSIS IN COMPLEX BANACH SPACES

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1. **Introduction.** Abstract spaces, and Banach spaces in particular, have played a prominent rôle in recent years in connection with many problems of analysis. There has also been a notable tendency for the concepts and tools of analysis to take a place alongside the algebraic and topological notions which are characteristic of the whole subject of abstract spaces. Thus analysis becomes more algebraic, and at the same time its range is broadened.

For a great many purposes it is immaterial whether a Banach space be real or complex. It is well known that a large portion of the theory of linear operations, as developed in Banach's book (Banach [1][1]), is equally valid for complex or real spaces. There are, however, situations in which the complex number system plays a crucial rôle. The theory of analytic functions in Banach spaces is a case in point. There are two large divisions of this subject: the theory of functions of a complex variable, the values of the functions lying in a Banach space, and the theory of analytic functions of an abstract variable. Our principal concern in this paper will be the first of these two theories. A brief survey of the second theory, and references to the literature, are given in §8.

Henceforth, except as otherwise stated, we shall use the term Banach space to mean a complex Banach space. The algebraic structure of such a space is that of an additive Abelian group with the complex numbers as operators. The topology of the space is defined by a *norm*; the norm of an element x is written $\|x\|$. It has the properties of an absolute value. Then $\|x-y\|$ is the distance between x and y, and the space is assumed to be a complete metric space.

2. **Analytic functions.** The basic development of the theory of analytic functions with values in a complex Banach space E, the independent variable being a complex number, follows the pattern of classical analysis. It seems to have been pointed out first by Wiener [1] that Cauchy's integral theorem is valid in this general setting. The usual consequences, such as the Cauchy integral formula, Liouville's theorem, and the Taylor and Laurent expansions, then follow. It is only when we come to theorems that deal in some way with di-

An address delivered before the Stanford meeting of the Society on April 24, 1943, by invitation of the Program Committee; received by the editors May 18, 1943.

[1] Numbers in brackets refer to the bibliography at the end of this paper.