THE BETTI GROUPS OF THE PRODUCT OF TWO NORMAL SPACES

C. E. CLARK

1. **Introduction.** Let R_1 and R_2 be homeomorphic to open sets of normal spaces. Furthermore, let each of R_1 and R_2 contain infinitely many points. For these spaces Alexandroff has defined inner Betti groups.[1] In this paper the inner Betti groups of the topological product $R_1 \otimes R_2$ are studied.

Let $\tilde{u}^r \in B^r(R_1)$, where B^r denotes the r-dimensional inner cohomology group with the integers as coefficient domain, and let $\tilde{u}^\rho \in B^\rho(R_2)$. To these two elements there corresponds an element $\tilde{u}^r \times \tilde{u}^\rho \in B^{r+\rho}(R_1 \times R_2)$. All such elements with $r+\rho=n$ generate a subgroup $B_1^n(R_1 \times R_2) \subset B^n(R_1 \times R_2)$. We characterize this subgroup (Theorem 3). In addition, we characterize the factor-group $B_2^n(R_1 \times R_2) = B^n(R_1 \times R_2)/B_1^n(R_1 \times R_2)$ (Theorem 4). In doing so we show that if $\tilde{u}^s \in B^s(R_1)$ is of order $\tilde{e}^s \neq 0$, and if $\tilde{u}^\sigma \in B^\sigma(R_2)$ is of order $\tilde{e}^\sigma \neq 0$, then to these two elements there corresponds an element $(\tilde{u}^s, \tilde{u}^\sigma) \in B_2^{s+\sigma-1}(R_1 \times R_2)$.

To prove these results we employ Alexandroff's second definition of the inner Betti groups which uses barycentric subdivisions of coverings.[2] Furthermore, Freudenthal's simplicial division of the product of two simplexes is used.[3]

2. **The groups** $B_i^n(K^a \times K^\alpha \bmod C^{a\alpha})$, $i=1, 2$. In this section we state without proof some facts about products of complexes which are consequences of [3]. Let K^a and K^α be finite complexes with subcomplexes C^a and C^α, respectively. Let B^n denote the n-dimensional integral cohomology group. To $u^r \in B^r(K^a \bmod C^a)$ and $u^\rho \in B^\rho(K^\alpha \bmod C^\alpha)$ there corresponds the product of these cohomology classes $u^r \times u^\rho \in B^{r+\rho}(K^a \times K^\alpha \bmod C^{a\alpha})$. We define $B_1^n(K^a \times K^\alpha \bmod C^{a\alpha})$ to be the subgroup of $B^n(K^a \times K^\alpha \bmod C^{a\alpha})$ generated by—we could say consisting of—all $u^r \times u^\rho$ with $r+\rho=n$. Let e^r and e^ρ be the orders of u^r and u^ρ, respectively, with the understanding that $e=0$ when u is free. Let (a, b) denote the greatest common divisor of a and b with the understanding that $(a, 0)=a$.

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[1] See [1, 7.2] (Numbers in brackets refer to references at end of paper).

[2] See [1, 9.22 and 9.4].

[3] See [2].