ON A THEOREM OF NEWSOM

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1. **Extension of the theorem.** In 1938, Newsom[1] published a paper containing a theorem regarding the behavior for large values of $|z|$ of the function

$$f(z) = \sum_{n=0}^{\infty} g(n)z^n, \tag{1}$$

radius of convergence equal to ∞. It is assumed that the function $g(w)$, where $w = x + iy$, satisfies the following two conditions:

(a) it is single-valued and analytic in the finite w-plane;

(b) it is such that for all values of x and y, one may write

$$| g(x + iy) | < Ke^{\pi |y|}, \tag{2}$$

where K is a positive constant and k is a positive integer. Under these conditions, according to the theorem, $f(z)$ may be expressed in the form

$$f(z) = \int_{-l-1/2}^{\infty} g(x) [\pm z]^x \frac{\sin k\pi x}{\sin \pi x} dx - \sum_{m=0}^{l} \frac{g(-m)}{z^m} + \xi(z, l), \tag{3}$$

where l is any positive integer, where the symbol $[\pm z]^x$ means z^x or $(-z)^x$ according as k is odd or even, respectively, and where if $|\arg [\pm z]| < \pi$, we have $\lim_{|z| \to \infty} z^l \xi(z, l) = 0$ for every value of l.

In the present paper we shall consider the situation when conditions (a) and (b) are made somewhat less restrictive. The theorem which we wish to prove is as follows:

THEOREM. *Let the coefficient $g(n)$ in* (1) *satisfy condition* (a) *except for a singularity at the point $w = w_1$, which is not a negative integer; and let inequality* (2) *be satisfied for all values of $|x|$ and $|y|$ sufficiently large. Then* (3) *continues to hold provided one subtracts from the right member the loop integral*

$$\frac{1}{2i} \int_C \frac{g(w)z^w}{e^{k\pi i w} \sin \pi w} dw \tag{4}$$

Received by the editors July 7, 1942.

[1] *On the character of certain entire functions in distant portions of the plane*, Amer. J. Math. vol. 60 (1938) pp. 561–572.