Geometry

329. Edward Kasner and J. J. DeCicco: *A generalized theory of dynamical trajectories.*

The differential geometry of the dynamical trajectories of positional fields of force has been completely developed in the *Differential-Geometric Aspects of Dynamics*, American Mathematical Society Colloquium Publications, vol. 3, 1913 (also Transactions of this Society, vols. 7–9 (1906–1910)). The fields of force, considered heretofore, depend only upon the position of the point. In their new work, the authors begin the study of the geometry of the generalized dynamical trajectories of fields of force which depend not only upon the position of the point but also upon the direction through the point. There are ∞^3 generalized dynamical trajectories. Any such system is completely characterized by the geometric Property I. The complete relationship between the rest trajectories and the lines of force is obtained (including the generalization of the theorem about one-third the curvature). Other systems of curves, such as velocity systems, systems S_k, pressure curves are defined for our generalized fields of force. These three new systems also possess the Property I. Complete geometric discussions of these systems are given. (Received August 24, 1942.)

330. Karl Menger: *Projective generalization of metric geometry.* I.

An F-projective space is called a set P if with each quadruple of elements p, q, r, s no three of which are identical an element (p, q, r, s) of a field F or ∞ is associated and $(p, q, r, s)=(q, p, s, r)=(r, s, p, q)$ and $(p, q, r, s)+(p, r, q, s)=(p, q, r, s)\cdot(p, s, r, q)=1$. An F-projective space, called $P_1(F)$, is obtained if for the elements of F the cross-ratio is defined as usual. In order that P be imbeddable into $P_1(F)$ it is necessary and sufficient that P satisfy the relation $(p, q, r, s)\cdot(p, s, r, t)=(p, q, r, t)$ and contain no pseudo-degenerate quadruple (four distinct elements with cross-ratio 0). The only non-imbeddable triples and pairs are pseudo-harmonic (that is, have cross-ratio -1). (Received August 3, 1942.)

331. Karl Menger: *Projective generalizations of metric geometry.* II.

P is called an $|F|$-projective space if with each quadruple p, q, r, s no three of whose elements are identical, a pair $(f, -f)$ of elements of F with the sum 0, denoted by $|p, q, r, s|$, is associated and $|p, q, r, s|=|q, p, s, r|=|r, s, p, q|$, $|p, q, r, s|\cdot|p, s, r, q|=|p, q, r, s|\cdot|p, r, s, q|\cdot|p, s, q, r|=1$, and $1+(f^2-g^2)^2=2(f^2+g^2)$ for $f=|p, q, r, s|$ and $g=|p, r, q, s|$. $P_1|F|$ is called the set F if $|p, q, r, s|=|(p, q, r, s)|$. In order that P be imbeddable into $P_1|F|$ it is necessary and sufficient that P satisfy the condition $|p, q, r, s|\cdot|p, s, r, t|=|p, q, r, t|$ and does not contain pseudo-degenerate quadruples or pseudo-harmonic triples or pairs, that is, distinct points with $|p, q, r, s|=|1|$ or not distinct points with $|p, q, r, s|=|2|$. (Received August 3, 1942.)

332. Saly R. R. Struik: *Emmy Noether and Maria Gaetana Agnesi, two outstanding women mathematicians.*

The work and life of these two mathematicians are evaluated and compared. (Received August 6, 1942.)