position fields coincide and are cyclic. The field  $\overline{L}$  is then equivalent to a subfield of  $\overline{K}'$ ; without loss of generality we may suppose  $\overline{K}' > \overline{L} \ge \overline{K}$ . The degree  $[\overline{L}:\overline{K}] = \overline{m}$  is a divisor of m. Consequently  $[Z_n\overline{L}:\overline{K}] = [Z_n\overline{L}:\overline{L}][\overline{L}:\overline{K}] = n\overline{m}$ . By the Galois theory there is then for every integer n an extension  $Z_n^*$  of degree n over  $\overline{K}$ . The defining equation  $f^*(x) = 0$  of  $Z_n^*/\overline{K}$  now may be approximated by an irreducible equation f(x) = 0 of degree n with coefficients in K so that  $Z_n^*$  is generated by the roots of f(x) = 0. The root field of f(x) = 0 over Kis the cyclic extension  $Z_n'$  of degree n over K. Hence  $Z_n^* = Z_n' \overline{K}$  for all n, contrary to the assumption that K is not relatively complete with respect to any rank one valuation.

HARVARD UNIVERSITY AND UNIVERSITY OF CHICAGO

## A DIFFERENTIAL GEOMETRY PROBLEM USING TENSOR ANALYSIS

## ATHERTON H. SPRAGUE

1. Introduction. The problem at hand was worked out in attempting to apply tensors to a much more general problem in classical differential geometry. The results obtained in a general coordinate system reduce readily to classical results of Eisenhart. An interesting interpretation of Christoffel symbols appears.

2. *R* net. A rectilinear congruence in 3-space is called a *W*-congruence if the asymptotic lines on the two focal surfaces correspond. If the tangents to both families of curves of a conjugate net on a surface form *W*-congruences the net is called an *R* net.<sup>1</sup> We derive the analytic conditions that must obtain in order that a given conjugate net on a surface shall be an *R* net.

3. Equations for an R net. Let  $S_1$  be one focal surface of a W-congruence, the vector equation of the surface being

(3.1) 
$$z_1^{\alpha} = z_1^{\alpha}(x^i), \qquad \alpha = 1, 2, 3; i = 1, 2.$$

1942]

Received by the editors November 11, 1941.

<sup>&</sup>lt;sup>1</sup> Tzitzeica, Comptes Rendus de l'Académie des Sciences, Paris, vol. 152 (1911), p. 1077.