## 266. Hermann Weyl: On Hodge's theory of harmonic integrals.

Hodge's fundamental existence theorem for harmonic integrals on Riemannian manifolds of any dimensionality is proved by the parametrix method. (The proof incorporated in Hodge's recent book on Harmonic Integrals, Cambridge, 1941, is wrong.) (Received July 1, 1942.)
267. Hassler Whitney: Differentiability of the remainder term in Taylor's formula.

If $f(x)$ is of class $C^{m}$, and $1 \leqq n \leqq m$, then $f(x)=\sum_{i=0}^{n-1} f^{(i)}(0) x^{i} / i!+x^{n} f_{n}(x) / n!$. It is shown that $f_{n}(x)$ is of class $C^{m-n}$, but not necessarily of higher class, and $\lim _{x \rightarrow 0} x^{k} f^{(m-n+k)}(x)=0(k=1, \cdots, n)$. A converse is true. A similar theorem holds in more dimensions. (Received July 28, 1942.)

## 268. Hassler Whitney: Note on differentiable even functions.

It is shown that an even function $f(x)$ of class $C^{2 s}$ (or class $C^{\infty}$, or analytic) may be written as $g\left(x^{2}\right)$, with $g$ of class $C^{s}$ (or class $C^{\infty}$, or analytic). (Received July 28, 1942.)
269. Hassler Whitney: The general type of singularity of a set of $2 n-1$ smooth functions of $n$ variables.

Let $f$ be a mapping of class $C^{1}$ of an $n$-manifold $M^{n}$ into an $M^{2 n-1}$. Then arbitrarily near $f$ is a mapping $f^{\prime}$, regular except at isolated singular points; at each of these, a certain condition (C) holds. (C) involves first and second derivatives, but is independent of the coordinate system employed. If (C) holds at $p$, and the mapping is of class $C^{4 r+8}$ (or class $C^{\infty}$, or analytic), then coordinate systems about $p$ and $f(p)$, of class $C^{r}$ (or class $C^{\infty}$, or analytic), exist such that the mapping is exactly $y_{1}=x_{1}^{2}, y_{i}=x_{i}$, $y_{n+i-1}=x_{1} x_{i}(i=2, \cdots, n)$. (Received July 28, 1942.)

## Applied Mathematics

270. Stefan Bergman: Operators in the theory of differential equations and their application. I.

By introducing $u=x \cos \theta+y \sin \theta, v=-x \sin \theta+y \cos \theta$ and $\xi=(\sigma / 2 k)+\theta$, $\eta=(\sigma / 2 k)-\theta$, where $\sigma_{x}=\sigma+k \sin 2 \theta, \sigma_{y}=\sigma-k \sin 2 \theta, \tau_{x y}=-k \cos 2 \theta$ the equations of the theory of plasticity can be written in the form $\left(\partial^{2} u / \partial \xi \partial \eta\right)-u / 4=0,\left(\partial^{2} v / \partial \xi \partial \eta\right)-v / 4$ $=0$ (see Geiringer and Prager, Ergebnisse der exakten Naturwissenschaften, vol. 13, p. 350). Here $\sigma_{x}, \sigma_{y}, \tau_{x y}$ are stresses, $x, y$, cartesian coordinates. Particular solutions of these equations can be written in the form $u(\xi, \eta)=\int_{-1}^{1} \exp \left(t(\xi \eta)^{1 / 2}\right)\left\{f\left[\xi\left(1-t^{2}\right) / 2\right]\right.$ $\left.+g\left[n\left(1-t^{2}\right) / 2\right]\right\}\left(1-t^{2}\right)^{1 / 2} d t$ where $f$ and $g$ are arbitrary twice continuously differentiable functions of one variable. (See Duke Mathematical Journal, vol. 6 (1940), pp. 538 and 557.) This class of functions possesses a base $\left\{u_{\nu}(\xi, \eta)\right\}$ such that each $u_{\nu}$ satisfies two (simple) ordinary linear differential equations of second order with rational coefficients. Entire solutions $u_{\nu}$ of the above partial differential equation are such that every $u$ defined in a convex domain can be approximated by sums of the form $\sum_{\nu=1}^{n} a_{\nu}^{(n)} u_{\nu}$. The author indicates an approximation procedure of a function $u$ given by its boundary values. These functions $u$ possess singularities which can be characterized in a way analogous to that in Comptes Rendus de l'Académie des Sciences, vol. 205 (1937), pp. 1360-1362. (Received June 3, 1942.)

