THEOREM 7. *The set* $\mathfrak{a}$ *is a modular ideal in* L *containing the set* M_L'.

PROOF. It is obvious that $\mathfrak{a}$ is an ideal; that it is modular follows from the corollary to Theorem 5, equations (7) and Lemma 2. Now if $(b, c)M'$, then by Lemma 2 $b\otimes c \notin L$, whence $bc=(b\otimes c)\otimes \infty$, and $bc\in\mathfrak{a}$. Hence $M_L'\subset\mathfrak{a}$.

It should be observed that both possibilities $M_L'=\mathfrak{a}$ and $M_L'\neq\mathfrak{a}$ can occur. In our special example where Λ is a projective geometry and $\Lambda - L$ consists of all subelements x of a hyperplane h with x not $\leqq\infty$, $M_L'=\mathfrak{a}$ if there exists $k\notin L$ with $\infty<k<h$. On the other hand, if no element exists (in Λ) between ∞ and h, then $M_L'\neq\mathfrak{a}$, for then $\infty\notin M_L'$. This example shows also that M_L' need not be an ideal.

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SUFFICIENT CONDITIONS THAT POLYNOMIALS IN SEVERAL VARIABLES BE POSITIVE

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1. **Introduction.** Let $T(x)$ denote a polynomial in a single real variable x with real coefficients. T. Popoviciu[1] established sufficient conditions that $T(x)$ be positive for all real x. In this paper we shall consider polynomials in several real variables with real coefficients, and establish sufficient conditions that the polynomials be positive for real values of the variables.

2. **Polynomials in two real variables.** In this section we shall develop sufficient conditions that a polynomial in two real variables with real coefficients be positive for all real values of the variables. Let us consider then,

$$T(x, y) = \sum_{i=0}^{2m} \sum_{j=0}^{2n} b_{ij}c_{ij}x^iy^j,$$

where the b_{ij} are positive constants, and the c_{ij} are real numbers. Introducing parameters, we write $T(x, y)$ in the following form

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[1] T. Popoviciu, *Sur une condition suffisante pour qu'un polynôme soit positif*, Mathematica, vol. 11 (1935), pp. 247–256.