A CHARACTERIZATION OF THE DISC

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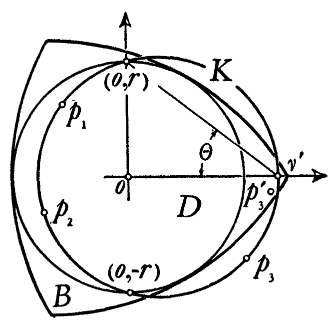
In this paper the disc[2] is characterized as the *only* connected, simply connected domain[3] B with the following property, C_3: B will cover (by an isometry) any subset P of the plane provided B will cover each 3 points of P.

The disc has property C_3. For a plane set P can be covered by a ρ-disc if and only if the members of the family F of ρ-discs with centers in P have a common point. If now each three points of P are



on a ρ-disc then each three discs of F intersect and by a theorem on convex bodies due to E. Helly[4] there is a point common to all the discs of F.

LEMMA. *A bounded, closed subset of the plane contains a largest circle.*

The proof is accomplished by selecting a sequence of circles from the set whose centers converge to a point and whose radii converge to the least upper bound of the radii of circles in the set and (using the fact that the set is closed) showing that the limiting circle belongs to the set.

THEOREM. *The disc is the only connected, simply connected domain with property* C_3.

PROOF. We assume B to be the given domain, and show that B

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[2] The disc of radius ρ and center p is the set of points x of E_2 such that $px \leqq \rho$.

[3] Closure of a bounded open subset of E_2.

[4] *Theorem.* If each $n+1$ sets of a family of *bounded*, closed, convex subsets of E_n intersect, there is a point common to all the sets. Jahrbuch der Deutschen Mathematiker-Vereinigung, vol. 32 (1932), pp. 175–176.