

relative to positive definite quadratic forms and Diophantine equations. For two incommensurable collinear vectors, the euclidean algorithm becomes the continued fraction algorithm, a powerful tool in approximation problems. It is conjectured that the generalized algorithm is as effective for problems of simultaneous approximation as the continued fraction algorithm is for simple approximation problems. To support this conjecture, satisfactory solutions of such problems are obtained by use of the generalized algorithm. (Received April 24, 1941.)

300. Albert Whiteman: *Sums connected with the partition function.*

The sum $A_k(n) = \sum \exp(\pi i s(h, k) - 2\pi i h n/k)$, where h runs over a reduced residue system with respect to the modulus k and $s(h, k)$ is a Dedekind sum, appears in Rademacher's formula for the number of partitions of n . On the basis of a different expression for this sum Lehmer (Transactions of this Society, vol. 43 (1938), pp. 271-295) factored the $A_k(n)$ according to the prime number powers contained in k , and evaluated the $A_k(n)$ in the case in which k is a prime or a power of a prime. A new approach to the first of these results has recently been given in a paper by Rademacher and Whiteman (American Journal of Mathematics, vol. 63 (1941), pp. 377-407). In the present paper the second of these results is derived by a method which is considerably simpler than Lehmer's. The paper also contains a new method for evaluating certain generalized Kloosterman sums. (Received May 29, 1941.)

ANALYSIS

301. R. P. Agnew: *On limits of integrals.*

It is shown that existence of $\lim_{A \rightarrow \infty} \int_A^{A+\lambda} f(t) dt$, for each λ in some set having positive measure, implies that the limit exists and is uniform over each finite interval of values of λ . The result is applied to prove two theorems of Iyengar (Proceedings of the Cambridge Philosophical Society, vol. 37 (1941), pp. 9-13) and the following Tauberian theorem. If $F(t)$ is absolutely continuous over each finite interval, if $\lim_{A \rightarrow \infty} \int_A^{A+\lambda} [F(t) - F'(t)] dt = 0$ for each real λ , and if $\lim_{t \rightarrow \infty} e^{-t} F(t) = 0$, then $\lim_{t \rightarrow \infty} F(t) = 0$. (Received April 11, 1941.)

302. Stefan Bergman: *A method for summation of series of orthogonal functions of two variables.*

Suppose $\{\Phi_\nu(\phi)\}$ [$(\phi) = (\phi_1, \phi_2)$] is a system of O.N. functions $[0 \leq \phi_k \leq 2\pi, \Phi_k(\phi_1 + 2\pi, \phi_2) = \Phi_k(\phi_1, \phi_2 + 2\pi) = \Phi_k(\phi_1, \phi_2), k = 1, 2], \sigma(e)$ a completely additive set function, $f(\phi_1, \phi_2) \in L^{1+p} (p > 0), a_\nu = \int_0^{2\pi} \int_0^{2\pi} \Phi_\nu d\sigma, b_\nu = \int_0^{2\pi} \int_0^{2\pi} \Phi_\nu f d\phi_1 d\phi_2$. Consider the series (1) $\sum_{\nu=0}^{\infty} a_\nu \Phi_\nu(\phi_1, \phi_2)$ and (2) $\sum_{\nu=0}^{\infty} b_\nu \Phi_\nu(\phi_1, \phi_2)$. Let \mathfrak{M} be a four-dimensional domain of the type described in Mathematische Annalen, vol. 104 (1931), pp. 611-636, with the distinguished boundary surface $\mathfrak{F} = E[z_k = h_k(\phi_1, \phi_2), k = 1, 2]$. Let $\Psi_\nu(z_1, z_2), \nu = 1, 2, \dots$, be the functions of the extended class which assume the values Φ_ν on \mathfrak{F} , and (3) $S(z_1, z_2) = \sum_{\nu=0}^{\infty} a_\nu \Psi_\nu(z_1, z_2)$, (4) $F(z_1, z_2) = \sum_{\nu=0}^{\infty} b_\nu \Psi_\nu(z_1, z_2)$. The series (4) converges absolutely and uniformly in every closed subdomain of \mathfrak{M} for $p > 1$. Using the results of Jessen, Marcinkiewicz and Zygmund (Fundamenta Mathematicae, vol. 25 (1935), pp. 217-234), Bergman and Marcinkiewicz (Fundamenta Mathematicae, vol. 33 (1939), pp. 75-94) and Bers (Comptes Rendus de l'Académie des Sciences, Paris, vol. 208 (1939), pp. 1273-1275 and 1475-1477) it is shown that S possesses a finite sectorial limit at the point $\{h_1(\phi), h_2(\phi)\}$ if σ possesses a finite strong derivative at (ϕ) , and F possesses the sectorial limit f almost everywhere on \mathfrak{F} . (Received May 2, 1941.)